Simulation of Three-Dimensional Dusty Plasmas

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Abstract—The structure and dynamics of dust particles in a 3-D dusty plasma is characterized using a Langevin molecular dynamics simulation with a Yukawa potential. Conditions are set appropriate for a liquid-like strongly coupled plasma. The positions of dust particles are shown in an image. The thermal motion of particles is decomposed into the longitudinal wave spectrum, showing a distinctive dispersion relation.

Index Terms—Dusty plasmas, liquids, numerical simulation, waves.

We use the Langevin dynamics simulation method of [1] to perform new simulations to characterize structure and waves in a 3-D dusty plasma. The equation of motion

\[ m_p \ddot{\mathbf{r}}_j = -\nabla \Phi_j - \frac{Q}{\lambda_D} e^{\frac{-|\mathbf{r}_j\cdot\mathbf{k}|}{\lambda_D}} \]

for the jth particle includes electrostatic forces due to particle–particle interaction \( \Phi_j \), a confinement potential \( \Phi \), gas friction \( -\nu_g m_p \dot{\mathbf{r}}_j \), and random Brownian force \( \zeta_j \) due to gas molecules. Here, \( m_p \) is the particle mass and \( \nu_g \) is the gas friction constant. We model the interparticle potential as a pairwise Yukawa potential, \( \phi_j(t) = (Q^2/4\pi\varepsilon_0|\mathbf{r}_j|) e^{-\frac{|\mathbf{r}_j\cdot\mathbf{k}|}{\lambda_D}} \)

for identical dust particles of charge \( Q \) and \( 2/3 \) is the Wigner-Seitz radius, \( n_D \) is the number density of dust particles, and \( T_D \) is the particle kinetic temperature. For the coupling parameter \( \Gamma > 1 \), the dust component is said to be strongly coupled, and dust particles can self-organize like atoms in a solid or liquid and sustain waves [2], [3].

Our simulation parameters are for the PK-4 instrument [4]. We use microsphere dust particles of radius 3.43 \( \mu \)m and \( m_p = 2.55 \times 10^{-13} \) kg, with neon gas at 50 Pa pressure and 0.03 eV temperature so that \( \nu_g = 51 \) s\(^{-1}\). We assume \( Q = -8520 e, n_D = 3 \times 10^4 \) cm\(^{-3}\), and \( \lambda_D = 8.3 \times 10^{-3} \) cm. The characteristic interparticle distance is \( a = 0.020 \) cm, so that \( \kappa = 2.4 \). The characteristic time for particle motion is \( \omega_p = 157 \) rad/s, where \( \omega_p = (Q^2 n_D e g m_p)^{1/2} \).

We chose \( T_D = 8.3 \) eV, corresponding to \( \Gamma \approx 63 \). For these values of \( \Gamma \) and \( \kappa \), the collection of dust particles is predicted to behave like a liquid, according to the phase diagram of the Yukawa system [5]. We simulate \( N = 12800 \) particles in a 3-D rectangular volume defined by a flat-bottomed confining potential \( \Phi \).

Results shown in Fig. 1(a) reveal the structural arrangement of the dust particles at a time during the simulation. This image was prepared by plotting the simulated particles in a 3-D coordinate system, with a sphere representing each particle. In this structure, each particle is in a cage defined by its nearest neighbors, but the structure is irregular, not crystalline. A video can be seen at [6] showing similar particles as shown in Fig. 1(a) from a rotating viewpoint.

To quantify the order of the 3-D structure, we calculate the pair correlation function \( g(r) \) [7], [8]. For this liquid, \( g(r) \) has only one distinctive peak in Fig. 1(b), indicating short-range translational order.

We characterize the dynamics using a wave spectrum. We start by using the particle position \( \mathbf{r}_j(t) \) and velocity \( \dot{\mathbf{r}}_j(t) \) to calculate the time series of the so-called longitudinal current, for a specified wave vector \( \mathbf{k} \)

\[ J_L(k, t) = N^{-1} \sum_{j=1}^{N} [\mathbf{r}_j(t) \cdot \mathbf{k}] \exp[\mathbf{r}_j(t) \cdot \mathbf{k}] \]

The spectral power \( |J_L(k, \omega)|^2 \) is then computed as the square modulus of the Fourier transformation in time of \( J_L(k, t) \).

Results in Fig. 1(c) show that, as expected, the spectral power is concentrated along a curved band. The band has a great width in the \( \omega-k \) space due to damping, arising from gas friction and the viscous motion of dust particles.

The band of spectral power in Fig. 1(c) corresponds to a real dispersion relation curve, which we plot in Fig. 1(c) as a dotted line. We determined this dispersion curve as the peak of the spectral power \( |J_L(k, \omega)|^2 \); to reduce the uncertainty, we computed \( \omega \) for the peak as the first moment of the spectral power \( |J_L(k, \omega)|^2 \) for each value of \( k \). The dispersion relation begins near \( ka = 0 \) with an acoustic-like upward slope, that is, the wave is forward for long wavelengths. This upward trend reverses for \( 2 < ka < 4 \), where the dispersion relation curve has a slightly negative slope, that is, the wave is backward.

REFERENCES

Fig. 1. (a) Structural arrangement of dust particles at a time during a 3-D Yukawa simulation. For clarity, the diameter of the dust particle microsphere is exaggerated here. Color indicates the height above the bottom plane. (b) Pair correlation function $g(r)$, indicating the disorder in the structural arrangement. (c) Wave spectrum for longitudinal motion, computed from particle positions and velocities. Color indicates spectral power, which is not smoothed; the results here have finite noise, which was reduced by averaging over eight runs of the simulation and 750 directions of $k$. Dotted curve: real dispersion relation, as obtained from the weighted peak of the spectral power with respect to frequency.