Nonlinear Wave Synchronization in a Dusty Plasma Under Microgravity on the International Space Station (ISS)

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***Abstract*— In a plasma containing micrometer-size dust particles, nonlinear wave synchronization was investigated experimentally under microgravity conditions on board the International Space Station (ISS). These dust particles were confined into a 3-D cloud, in the vicinity of a diffuse edge of the plasma, which was generated by an inductively coupled radio frequency (RF) glow discharge. A cross-sectional slab of the cloud was imaged using a video camera. The dust-density fluctuations in the slab were characterized using the video image data. A steady and long-lived dust acoustic wave (DAW) was observed to be spontaneously generated in the cloud; it propagated through the dust cloud, which had a gradually varying density distribution. Two kinds of spectral analyses of the wave motion were performed, using Fourier transforms and Hilbert transforms, respectively; these revealed two distinctive spatial domains in the cloud, termed frequency clusters. Within each cluster, waves were found to oscillate at a dominant frequency that remained constant, manifesting mutual synchronization throughout the cluster. Across the two clusters, the dominant frequency exhibited a step-wise change, with a frequency ratio of 2:1, which is consistent with phase-lock conditions for a harmonic synchronization state.**

***Index Terms*— Dust acoustic wave (DAW), dusty plasma, microgravity, PK-4, synchronization.**

I. INTRODUCTION

SYNCHRONIZATION is a nonlinear phenomenon widely observed in biological, chemical, and physical systems [1]–[3]. It can arise, for example, in an ensemble of nonlinearly coupled oscillators. Due to nonlinear interactions, these oscillators can cooperatively adjust their rhythms or frequencies, and they become fully or partially synchronized with each other [1], [2]. When partially synchronized, the ensemble exhibits clusters of oscillators that vibrate at constant frequencies, that is, frequency clustering [4]. Here, we investigate the nonlinear wave synchronization and frequency clustering in a plasma that contains micrometer-size dust particles, that is, a dusty plasma.

A dusty plasma is a four-component mixture of electrons, ions, neutral gases, and micrometer-size dust particles [5]–[10]; the dust component is of interest here. In a typical glow-discharge plasma, dust particles are charged negatively due to plasma electrons. They mutually repel each other, sustaining various waves to propagate in the plasma. A low-frequency compressional density wave called dust acoustic wave (DAW) [11]–[13] is particularly of interest because it can be spontaneously self-excited, due to an instability arising from flowing ions [14]–[35] in the plasma. The DAW often grows to such a large amplitude that nonlinear phenomena like synchronization can occur.

The synchronization of DAWs was previously studied in several dusty plasma experiments. Menzel *et al.* [28]–[30] discovered frequency clustering in an experiment performed in a capacitively coupled parallel-plate radio frequency (RF) reactor; this experiment was performed under microgravity conditions using parabolic flights. Williams [32] investigated the time evolution of frequency clusters in a dc glow-discharge plasma. Killer and Melzer [35] used a capacitively coupled RF plasma to demonstrate that strongly nonlinear DAWs can exhibit global coherence with high levels of synchronization. Aside from these studies, where the nonlinear interactions were among the waves themselves, there has also been a demonstration that a nonlinear interaction of a DAW with an externally driven sinusoidal modulation of the plasma can cause the DAW to synchronize with the modulation [31], [34].

In this article, we report a microgravity experiment using the PK-4 instrument [36]–[40] on the International Space Station (ISS), investigating the nonlinear wave synchronization in 3-D

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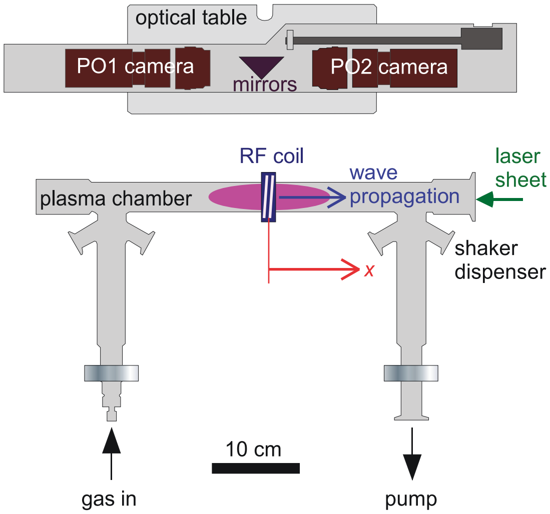

Fig. 1. Sketch of the experimental setup using the PK-4 instrument on the ISS. In a glass-tube plasma chamber, a glow-discharge plasma was generated using an inductive RF coil powered at RF. Using a shaker dispenser, dust particles were injected into the plasma, forming a dust cloud in the vicinity of a diffuse edge of the plasma. Two PO cameras then viewed a cross-sectional slab of the dust, which was illuminated by a laser sheet. Waves were observed to be excited spontaneously; they propagated in the $+x$-direction.

dusty plasmas with micrometer-size particles. Our plasma was different from that in previous synchronization experiments; the RF power was inductively coupled, and the chamber was a glass tube. Our previous work in this plasma [40] has shown that it allows the spontaneous excitations of long-lived DAWs. Here, we will explore a nonlinear feature of the DAWs, a frequency clustering due to mutual synchronization among adjacent constituent elements in the dust cloud.

II. EXPERIMENTAL

This microgravity experiment was carried out during the PK-4 Science Campaign 5, on orbit in the ISS. This article is based on data from the same experiment as in a previous article [40], which centered on different scientific topics, the decay of DAW modes as characterized by correlation functions, the usefulness of linear dispersion relation theory, and the spectrum of DAWs. The present article is based on different experimental runs (i.e., recorded at different times), as compared to [40]. Our purpose here is to find indications of nonlinear synchronization.

Here, we briefly review our experiment, which was performed using the joint ESA-Roscosmos facility Plasma kristall-4 (PK-4) on the ISS [36]. Further details are presented in [40]. Fig. 1 is a sketch of the PK-4 instrument, showing some features used for the present investigation. In a U-shaped glass tube, an inductively coupled RF glow plasma was generated by ionizing neon gas at the pressure 35 Pa. The glow discharge was sustained by the RF coil at 80 MHz with 0.4 W of power. A dust cloud was confined in the vicinity of the diffuse edge of the plasma to the right of the RF coil in Fig. 1. The dust particles had 6.86-μm diameter and 2.55×10^{-13}-kg mass. A cross-sectional slab of the dust cloud was illuminated by a laser sheet; it was imaged using two particle observation (PO) cameras. Video image data were recorded at 100 frames/s, with a resolution of 0.0142 mm/pixel. An example image is shown in Fig. 2(a), where white dots represent images of individual particles.

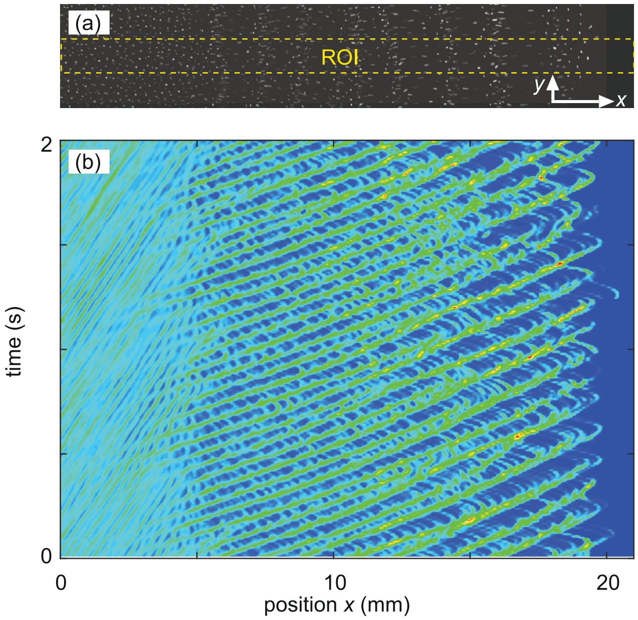

Fig. 2. (a) Snapshot image of a cross-sectional slab of dust, viewed from a PO camera. White dots represent individual particles, while the spatial patterns with the length scale ≈2 mm are due to waves propagating in the $+x$-direction. We analyze the variation of the pixel intensity inside the marked rectangular region, ROI, for consecutive video frames, yielding the space–time diagram in (b). Each color level in (b), at specified x and t, represents a pixel intensity that was averaged over y in ROI, for the frame corresponding to t. The sloped stripes at large x in (b) represent wavefronts, while the stripes at $x < 5$ mm are mainly due to a drift motion in the dust. Note that wavefronts merging is seen in (b), for $10 < x < 20$ mm.

We analyzed the image data, yielding a measurement of the spatial and temporal evolution of dust density. Here, the density is indicated by brightness in the image. For each image, we chose a rectangular region of interest, the middle one-third of the image, marked ROI in Fig. 2(a). We computed a mean pixel intensity for each x position in the ROI by averaging over the pixels in a vertical bin of one-pixel width. This averaging yielded a spatial variation of pixel intensity with x, for one frame. Repeating the above averaging for 200 consecutive video frames yielded a space–time diagram as shown in Fig. 2(b), for one experimental run. In total, we obtained the space–time-diagram data for four experimental runs.

We performed spectral analysis of the space–time-diagram data two ways, using a Fourier transform and a Hilbert transform. Both transforms have an input of spatiotemporal data, but the Hilbert transform is better suited for identifying a distinctive frequency, which is termed the “instantaneous frequency” [29].

The Fourier analysis was essentially a transformation of the dust-density-fluctuation data as in Fig. 2(b) from time domain to frequency domain. It yielded the power spectra shown as color contours in Fig. 3(a), which has been averaged over four experimental runs. This power spectrum is presented as a function of position.

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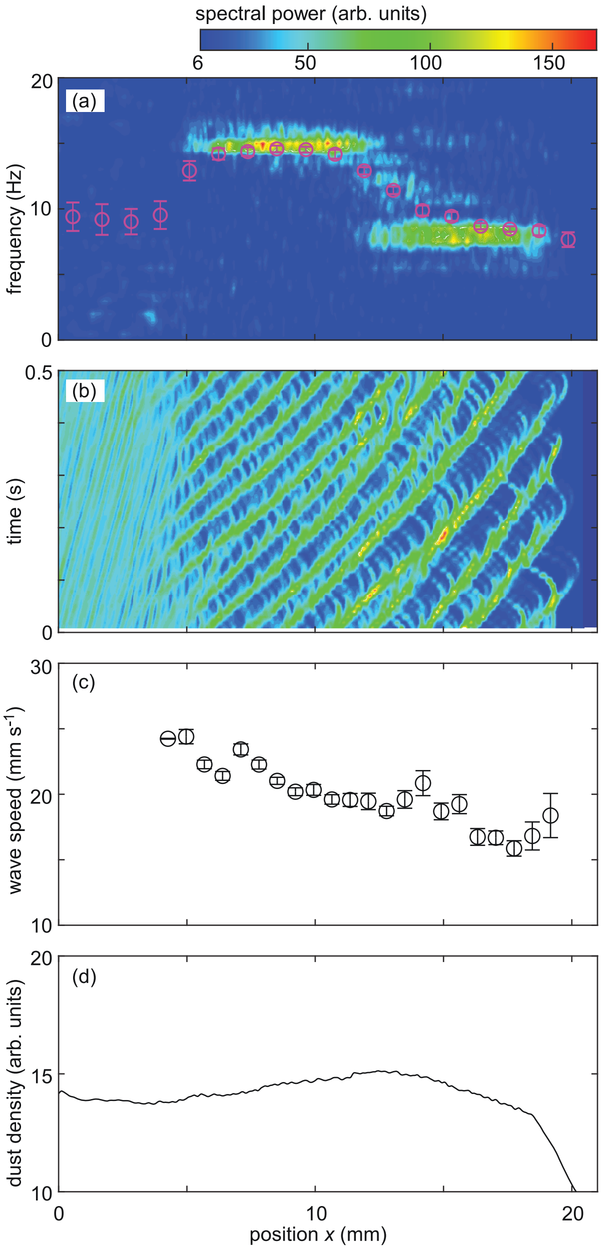

Fig. 3. (a) Identifying frequency clustering, using power spectra (the contour plot) and instantaneous frequencies (the scattered data points) of dust motion. The spectra power data were obtained from a Fourier transform of the space–time-diagram data as in Fig. 2(b), while the instantaneous-frequency data were from a Hilbert analysis of the same diagram data. Spectral concentration is seen at two dominant frequencies: 15 and 7.5 Hz, corresponding to two frequency clusters. The clusters exhibit frequency plateaus, due to mutual synchronization. The ratio of the two dominant frequencies, 2:1, is consistent with the phase-lock conditions for a harmonic synchronization state. (b) Portion of the space–time diagram shown in Fig. 2(b). (c) Wave speed obtained from a linear fit of the wavefronts in the space–time diagram as in Fig. 2(b). The speed exhibits a downward but gradually varying trend with x. (d) Spatial profile of dust density, obtained by averaging the space–time-diagram data over time. The dust cloud was nonuniform, but with a rather gradually varying density gradient.

The Hilbert analysis [29] of the same space–time-diagram data yielded the data points in Fig. 3(a), which show the variation of instantaneous frequency with x. The Hilbert analysis was done in time domain. First, an instantaneous phase was computed from the analytic signals [29] obtained by Hilbert-transforming the space–time-diagram data. Second, an instantaneous frequency was computed by a linear regression of the phase data with time, at each position x. Third, three averages were performed for the instantaneous-frequency data, yielding a mean frequency and its uncertainty. We averaged the instantaneous-frequency data for every 16 adjacent pixel positions in x, yielding a frequency variation with x for each experimental run. Then we averaged the frequency variation for four experimental runs, yielding a mean frequency and its standard deviation. Finally, for the purpose of a better presentation, we further reduced the number of data points by performing a weighted average over every four data points from the previous average, with the standard deviation as weights. This resulted in the instantaneous frequency in Fig. 3(a), shown as scattered data points.

In general, we found an agreement between the results from the spectral analysis using the Hilbert and Fourier transforms. For example, with the region $8 < x < 11$ mm, the data points from the Hilbert transform mostly fall within the strongest band of spectral power from the Fourier transform. Further comparisons of the two kinds of transforms are presented in Section III.

The data that we will emphasize in our analysis are in the region $x > 5$ mm. For smaller values of x, although our Hilbert analysis yielded instantaneous-frequency data, the wave amplitude was too weak to yield a strong signature in the Fourier power spectrum shown as color contours. Moreover, for $x < 5$ mm, the dust had a visible drift motion, which could affect the accuracy of our analysis.

III. RESULTS

Here, we discuss our results for DAWs in the context of synchronization. These waves were self-excited, due to flowing ions in an inductively coupled RF glow plasma. In [40], based on the same experiment on the ISS, we investigated linear features of the waves, including wave's spectrum and dispersion relation. Results presented here will reveal some nonlinear features of these waves, including indications of synchronization.

Our analysis was based on the spatial and temporal variation of dust density. The space–time diagrams in Figs. 2(b) and 3(b) show an example of the variation, for an experimental run of 2-s duration. Wavefronts are seen in Figs. 2(b) and 3(b), for $x > 5$ mm; they propagate in the $+x$-direction.

Fig. 3(c) shows a wave speed as a function of position x. This speed was obtained by fitting the slope of wavefronts like those in Figs. 2(b) and 3(b). The wave speed exhibits a downward trend, with increasing position x, but this variation is gradual.

Fig. 3(d) shows the spatial profile of dust density, which was obtained by averaging the space–time data over time. The profile indicates that our dust cloud had a nonuniform density, but without any steep gradients in the main region that was analyzed.

We draw attention especially to a prominent feature in the power spectra in Fig. 3(a). There is a concentration of power in the frequency domain. In Fig. 3(a), this concentration can

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be seen from the two horizontal stripes, which are localized into two frequency bands at the dominant frequencies: 15 and 7.5 Hz, respectively. This spectral concentration corresponds to the periodic motion as seen in Figs. 2(b) and 3(b).

We interpret this concentration of spectral power, in two frequencies at different positions, indicating two spatial frequency clusters in our dust cloud. The term cluster here refers to a spatial region within which the wave frequency remains constant. From Fig. 3(a), the two clusters were formed in the regions $5 < x < 13$ mm and $13 < x < 20$ mm, corresponding to the two dominant frequencies 15 and 7.5 Hz, respectively. The frequency ratio 2:1 for the two clusters is consistent with the phase-lock conditions for a harmonic synchronization state.

The formation of the clusters, or frequency clustering, is an indication of nonlinear wave synchronization. Within a cluster, the wave exhibited a constant frequency or frequency plateau, even though our dust cloud was nonuniform, as indicated by Fig. 3(d). Furthermore, the wave frequency exhibited a step-wise change across the clusters, even though such radical change is lacking in the density profile Fig. 3(d). For nonuniform linear systems, such frequency plateau and step-wise changes are not expected, given the dust density profile as in Fig. 3(d) and the wave speed in Fig. 3(c). However, for our experimental system with nonlinear DAWs, frequency plateaus within a cluster and frequency transition across the clusters are signatures of mutual synchronization. The plateau arose when all the adjacent constituent elements in a cluster were nonlinearly coupled and synchronized to a dominant frequency. The step-wise frequency distribution arose because the coupling was not strong enough to have a global synchronization, and therefore the synchronization was partial. Signatures like these have been predicted theoretically for dusty plasmas, modeling the spatiotemporal character of the DAWs using a chain of nonlinear van der Pol oscillators that are self-sustained and mutually coupled [30].

Other features visible in Fig. 3(a) include some scattering of spectral content outside the two frequency bands. This scattering is noticeable over a range of positions, particularly near the transition region of the two bands. In this transition region, we can also identify wavefront merging in Figs. 2(b) and 3(b). The occurrence of the merging can linger well beyond the transition region.

The instantaneous frequency, as obtained by the Hilbert transform, generally followed the trends of the dominant frequencies. In particular, the instantaneous frequency data match that for the dominant frequency of the cluster at $5 < x < 13$ mm. This agreement coincides with the absence of significant spectral content scattering outside the dominant frequency band at 15 Hz.

We find that the instantaneous frequency did not exhibit the same step-wise change as seen from our Fourier-transform power spectrum. For the cluster with 7.5-Hz dominant frequency, the data points for the instantaneous frequency approach gradually to the frequency band in the spectra, for $10 < x < 15$ mm. We note that the instantaneous-frequency data were obtained from the phase data in the time domain, so that the results for a given position are condensed into a single number, a mean frequency, reflecting all possible spectral content. The Fourier transform analysis is presented as a function of frequency, so that one can identify not only peaks in the spectral power, but also weaker spectral content that has been scattered outside the two frequency bands, which is apparent in Fig. 3(a). That scattered content contributes to the instantaneous frequency in the Hilbert transform analysis.

We speculate that the gradual transition in the instantaneous frequency might be caused by wavefronts merging. From Fig. 2(b), some wavefronts travel far into the right cluster without merging, which has the effects of reducing the frequency.

Finally, we note that our experiment is yet another confirmation of mutual synchronization in dusty plasmas. Frequency clusters were first observed by Menzel *et al.*, in a capacitively coupled RF plasma on a parabolic flight [28]. They were also investigated in a dc discharge plasma [32] on ground-based conditions. Our experiment is different from the previous experiments. Our plasma was generated differently, in an inductively couple RF glow discharge, and our experiment was performed on board the ISS. Microgravity on the ISS allowed a large and long-lived 3-D dust cloud for the experiment.

IV. CONCLUSION

We experimentally investigated the nonlinear wave synchronization in a 3-D dusty plasma under microgravity, using the joint ESA-Roscosmos facility PK-4 on the ISS. A dust cloud was confined in an inductively coupled RF-discharge glow plasma in low-pressure neon; it exhibited self-excited DAWs, which were nonlinear due to energy input from flowing ions in the plasma. A Fourier spectral analysis of the spatial and temporal variations of these density waves was performed, revealing the formation of two distinctive spatial frequency clusters. Each cluster was found to oscillate at a constant dominant frequency, due to mutual synchronization. The frequencies for the two clusters had a ratio of 2:1, which corresponds to the phase-lock conditions of a harmonic synchronization state. Hilbert-transform analysis yielded instantaneous frequencies of dust motion, which are consistent with the dominant frequencies from the Fourier spectral analysis.

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