# Effect of strong coupling on the dust acoustic instability

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In a plasma containing charged dust grains, the dust acoustic instability (DAI) can be driven by ions streaming through the dust with speed less than the ion thermal speed. When the dust is strongly coupled in the liquid phase, the dispersion relation of the dust acoustic modes changes in a way that leads to an enhancement of the growth rate of the DAI. In this paper, we show how strong coupling enhances the DAI growth rate and consider application to microgravity experiments where subthermal ion flows are in general possible.

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## I. INTRODUCTION

Dusty plasmas are plasmas containing small (micron to submicron) sized dust grains that become electrically charged in the plasma owing to various processes including the collection of plasma electrons and ions. In low-temperature laboratory or microgravity experiments the dust is negatively charged because of the higher mobility of the electrons. In these environments the ensemble of grains can often be strongly coupled, which means the electrostatic interaction energy between neighboring grains is much larger than the thermal (kinetic) energy of the grains. Such systems are usually modeled as Yukawa systems, that is, a system of charged particles interacting via a screened Coulomb interaction, with the screening provided by the background plasma. The parameters that generally characterize the strength of the electrostatic coupling between dust grains are the bare Coulomb coupling parameter  $\Gamma = Z_d^2 e^2 / aT_d$  and the screening parameter  $\kappa =$  $a/\lambda_D$ . Here  $Z_d$  is the dust charge state,  $T_d$  is the thermal (kinetic) energy of the dust particles, a is the Wigner-Seitz radius, which is related to the dust number density  $n_d$  by  $a^3 = 3/4\pi n_d$  for a three-dimensional (3D) system, and  $\lambda_D$ is the plasma screening length. When the effective screened Coulomb (Yukawa) coupling parameter, roughly  $\Gamma_d = \Gamma e^{-\kappa}$ , is  $\gg1$  but smaller than that required for crystallization, the system is in the strongly coupled liquid phase.

A number of theoretical studies have shown how strong coupling affects the dispersion relation of dust acoustic waves (DAWs) [1] in dusty plasmas in the liquid phase (see reviews in, e.g., Refs. [2–5] and references therein). Strong coupling affects the DAWs via a reduction of the phase speed and maximum frequency relative to the value they would have with the same density, charge and mass in the weak coupling approximation, and the onset of negative dispersion (i.e.,  $\partial\omega/\partial k < 0$ ) at shorter wavelengths [6,7]. Here we consider the effect of strong coupling on the dust acoustic instability

(DAI) in a 3D dusty plasma. In the scenario considered here, the DAI is driven by ions streaming through the dust with speed less than the ion thermal speed [8]. We extend our preliminary analysis of the effect of strong coupling on this instability [9] by including dust collisional effects, and by considering application to microgravity experiments where dust wave activity has been reported (e.g., Refs. [10-12]) and where subthermal ion flows are in general possible. (The effect of strong coupling on an ion-dust streaming instability when the ions are treated in the fluid approximation was considered to some extent in Refs. [13–15].) Section II outlines the analysis, which uses the quasilocalized charge approximation (QLCA) [16]. Section III compares our results with experimental observations of DAWs in microgravity experiments that may have subthermal ion flows. Section IV gives a brief summary and discussion.

## **II. ANALYSIS**

The negatively charged dust grains will be assumed to interact via a Yukawa interaction, with the screening provided by the background plasma. The plasma is charge neutral, with the equilibrium charge neutrality condition

$$n_e + Z_d n_d = n_i, \tag{1}$$

where  $n_j$  and  $Z_j$  are the number density and charge state of particle species j, with the subscripts e, i, and d denoting electrons, ions, and dust, respectively. We use the QLCA (for details, see Refs. [16,17]) to treat the effects of strong coupling on the DAI, assuming the dust grains are in the strongly coupled liquid phase. The QLCA is based on the premise that the quasilocalization of the strongly coupled particles governs the formation of the collective modes. In this approach, the dispersion of the longitudinal modes in the strongly coupled dusty plasma is described by [6,7]

$$\epsilon_L(k,\omega) = 1 + \sum_j \alpha_j = 0, \qquad (2)$$

where  $\alpha_i$  are the susceptibilities of the charged particles.

In our model plasma, the ions and electrons are assumed to flow with subthermal speeds relative to the dust due to an external electric field. Taking into account collisions, using a number conserving Krook collision operator, and assuming one-dimensional propagation along the direction of ion streaming, the susceptibility of the weakly coupled electrons and ions would be given in standard plasma theory by [18–20]

$$\alpha_j = \frac{1}{k^2 \lambda_{Dj}^2} \frac{[1 + \zeta_j Z(\zeta_j)]}{[1 + (i\nu_j/\sqrt{2}k\nu_j)Z(\zeta_j)]},$$
(3)

where

$$\zeta_j = \frac{\omega \mp k V_{0j} + i \nu_j}{\sqrt{2}k \nu_j}$$

where the upper (lower) sign refers to ions (electrons). Here the Debye length  $\lambda_{Dj} = (T_j/4\pi n_j Z_j^2 e^2)^{1/2}$  where  $T_j$  is the temperature,  $v_j = (T_j/m_j)^{1/2}$  is the thermal speed with  $m_j$ being the mass,  $v_j$  is the collision frequency,  $\mathbf{V}_{0j}$  is a streaming velocity, and  $Z(\zeta)$  is the plasma dispersion function [21]. The phase speed of a DAW is generally orders of magnitude smaller than the ion thermal speed in laboratory dusty plasmas. We consider the kinetic limit  $\zeta_j \ll 1$ , which also implies that the mean free path for electron and ion collisions is much larger than the wavelength of the DAW. Expanding the plasma dispersion function in (3) and neglecting collisional corrections, we have simply for the electrons and ions,

$$\alpha_j \approx \frac{1}{k^2 \lambda_{Dj}^2} \left[ 1 + i \sqrt{\frac{\pi}{2}} \frac{\omega \mp k V_{0j}}{k v_j} \right]. \tag{4}$$

In (4), the real, static response is responsible for the screening of the dust grains, and will be absorbed into the dust susceptibility in order to model dust interacting via a Yukawa interaction. The imaginary part of (4) includes the subthermal streaming that can drive wave growth.

We assume that the dust, which interacts via a Yukawa interaction, is cold and stationary, and that the dust susceptibility can be modeled as a Drude model with strong coupling (see, e.g. [6,7],). Then the dispersion relation (2) can be written as

$$1 + i \sum_{j=e,i} \frac{\lambda_D^2}{\lambda_{Dj}^2} \sqrt{\frac{\pi}{2}} \frac{(\omega \mp k V_{0j})}{(1 + k^2 \lambda_D^2) k v_j} - \frac{\Omega_{L0}^2}{\omega(\omega + i v_d) - D_L(k)} \approx 0.$$
(5)

Here  $\Omega_{L0}$  is the longitudinal DAW frequency in the weakly coupled phase,  $\Omega_{L0} = k\lambda_D \omega_{pd}/(1 + k^2 \lambda_D^2)^{1/2}$ , where  $\omega_{pd} = (4\pi Z_d^2 n_d/m_d)^{1/2}$  is the dust plasma frequency,  $m_d$  is the dust mass, and  $\lambda_D = (\lambda_{Di}^{-2} + \lambda_{De}^{-2})^{-1/2}$ . In addition,  $\nu_d$  is the dust collision frequency, and the upper (lower) sign in the summation term in (5) refers to ions (electrons). The effect of strong coupling appears in the Drude model for the dust via  $D_L(k)$ , the longitudinal projection of a dynamical matrix, akin to that in the harmonic theory of lattice phonons [3]. It is given by [3,7]

$$D_L(k) = -\frac{k_\mu k_\nu}{k^2} \frac{n_d}{m_d} \int d^3 r [\partial_\mu \partial_\nu \phi(r)] (e^{i\mathbf{k}\cdot\mathbf{r}} - 1)h(r),$$

where  $\phi(r) = (Z_d^2 e^2 / r) e^{-r/\lambda_D}$  is the Yukawa potential and h(r) is the equilibrium pair correlation function. (Explicit expressions for  $D_L(k)$  for a Yukawa potential are given in Refs. [3,7].) Because it is proportional to the correlation energy of the strongly coupled particles for small k, the function  $D_L(k)$ is <0. We will use a local field function  $D_L(k)$  for arbitrary k, which is computed numerically as a functional of the equilibrium pair correlation functions obtained from molecular dynamics simulations for a given combination of  $\Gamma$  and  $\kappa$ . The neglect of dust thermal effects in (5) is justifiable since  $\Gamma \gg 1$ in the strongly coupled phase. While thermal effects can affect wave dispersion in the liquid phase (see Refs. [22,23]), these effects are more important in the Vlasov (weakly coupled) phase where thermal effects can lead to an increase in the DAW frequency at larger k as well as dust Landau damping. We will neglect thermal effects for simplicity, in order to focus on how strong coupling affects the DAI.

In the following section, we will numerically solve (5). Here we show the form of the solution for  $v_d = 0$ . We assume that  $T_e \gg T_i$ , as is generally the case in laboratory dusty plasmas, so that the linearized Debye length  $\lambda_D \approx \lambda_{Di}$ . In addition, we consider that the instability is driven by ion streaming, with  $V_{0i}/v_i \gg V_{0e}/v_e$ . The latter condition often holds in laboratory dusty plasmas where the streaming is due to an electric field (ions and electrons stream in opposite directions) and the stream speed is given by balancing the electrostatic force with the drag force due to collisions with neutrals. Taking  $V_{0i} \gg$  $\omega/k$  and  $\omega = \omega_r + i\gamma$ , where  $|\gamma| \ll \omega_r$ , the real and imaginary parts of the frequency are obtained as (see also Ref. [24])

$$\frac{\omega_r^2}{\omega_{nd}^2} \approx \frac{k^2 \lambda_D^2}{1 + k^2 \lambda_D^2} + \frac{D_L(k, \Gamma, \kappa)}{\omega_{nd}^2},\tag{6a}$$

$$\frac{\gamma}{\omega_{pd}} \approx \sqrt{\frac{\pi}{8}} \frac{k^2 \lambda_D^2}{\left(1 + k^2 \lambda_D^2\right)^2} \frac{V_{0i}}{v_i} \frac{\omega_{pd}}{\omega_r}.$$
 (6b)

From (6), noting that  $D_L < 0$ , we see that strong coupling leads to a decrease in the dimensionless real frequency  $\omega_r/\omega_{pd}$ , which suggests that the growth rate, which is inversely proportional to  $\omega_r/\omega_{pd}$ , increases as compared with the case when the dust is weakly coupled. [Note that for constant  $Z_d$ and  $n_d$  (i.e.,  $\omega_{pd}$ ), an increase in coupling corresponds to a decrease in the dust thermal energy.] The physical reason the growth rate increases may be that, as the frequency of the dust acoustic wave decreases due to strong coupling, a larger portion of the ion velocity distribution could participate in inverse ion Landau damping, which drives this instability. We should also point out that (6) was obtained assuming the dust is cold: if dust thermal effects were included, the growth rate could be reduced due to dust Landau damping effects.

#### **III. NUMERICAL RESULTS**

First we illustrate how strong coupling can increase the growth rate of the DAI, neglecting dust collisional effects. Figure 1 shows the real and imaginary parts of the frequency



FIG. 1. (Color online) (a) Real frequency  $\omega_r$  and (b) growth rate  $\gamma$  normalized to  $\omega_{pd}$  versus  $k\lambda_{Di}$  obtained by solving (5). Parameters are  $v_d = 0$ ,  $T_e/T_i = 100$ ,  $n_i/n_e = 2$ ,  $V_{0i}/v_i = 0.2$ ,  $V_{0e}/v_e = 0$ , and  $\omega_{pd}/\omega_{pi} = 1 \times 10^{-4}$ . With strong coupling,  $\Gamma = 725$ ,  $\kappa = 3$  (magenta, dashed curves),  $\Gamma = 300$ ,  $\kappa = 2$  (red, solid curves), and the weakly coupled fluid case, obtained by setting  $D_L(k) = 0$  (black, dot-dash curves).

obtained by solving (5) for the following parameters:  $v_d = 0$ ,  $V_{0i}/v_i = 0.2$ ,  $V_{0e}/v_e = 0$ ,  $T_e/T_i = 100$ ,  $n_i/n_e = 2$  and  $\omega_{pd}/\omega_{pi} = 1 \times 10^{-4}$ . The solid and dashed curves show results with strong coupling. For comparison, the dot-dashed curve shows the behavior obtained by setting  $D_L(k) = 0$ , which we refer to as the weakly coupled fluid case, appropriate for cold, weakly coupled dust.

Next we include dust collisional effects in an effort to consider possible application of our results to microgravity experiments where dust waves have been observed (e.g., Refs. [10,12]). In microgravity experiments, subthermal ion flows are in general possible. Though the waves observed may be nonlinear, linear theory should give conditions for the onset of self-excited waves [11]. The motivation here is to see if the inclusion of strong coupling effects would predict a DAI even when it would be quenched by dust collisional damping in a weakly coupled fluid model.

First we consider the microgravity experiments reported by Arp *et al.* [10]. The following set of parameters may be roughly representative of those given in relation to Fig. 3 in Arp *et al.* [10] : argon pressure  $P \sim 15$  Pa, density of argon ions  $n_i \sim 2 \times 10^8$  cm<sup>-3</sup>,  $T_e \sim 4$  eV,  $T_i \sim 0.026$  eV, dust radius  $R \sim 3.4 \,\mu\text{m}$ , and an average distance *d* between grains of about  $d \sim 270 \,\mu\text{m}$ . With these values, the ion Debye length is estimated to be about  $\lambda_{Di} \sim 85 \,\mu\text{m}$ . In addition, assuming that the Wigner-Seitz radius  $a \sim (3/4\pi)^{1/3}d$  we estimate  $n_d \sim 5 \times 10^4$  cm<sup>-3</sup>. Using a dust mass density of about 1.5 g/cm<sup>3</sup>, the dust mass is estimated to be about  $m_d \sim 1.4 \times 10^{14}$  times the proton mass  $m_p$ . To estimate the dust charge state, we note that standard orbit-motion-limited (OML) theory (see, e.g., Ref. [25]) would give too large a value for the dust charge, with the  $Z_d n_d > n_i$ . Thus there may be electron depletion effects (see, e.g., Ref. [25]) to limit the dust charge to  $Z_d < 4000$ . (It should also be pointed out that OML can overestimate the grain charge state when there are significant ion-neutral collisions [11].) Thus we use a nominal value of  $Z_d \sim 3500$ . Then  $\omega_{pd} \sim 87$  rad/s and the ratio of the dust to ion plasma frequencies is  $\omega_{pd}/\omega_{pi} \sim 3 \times 10^{-5}$ . The ion-neutral and electron-neutral collision frequencies are modeled as  $v_j \sim \sigma_{jn} n_n v_j$ , where j = e,i for electrons and ions, respectively,  $\sigma_{jn}$  is the cross section for collisions with neutrals, and  $n_n$  is the neutral density. Using  $\sigma_{in} \sim 5 \times 10^{-15}$  cm<sup>2</sup> and  $\sigma_{en} \sim 5 \times 10^{-16}$  cm<sup>2</sup> we have that  $v_i/\omega_{pi} \sim 0.16$  and  $v_e/\omega_{pi} \sim 53$ . The dust-neutral collision frequency is given by

$$v_d = \eta \frac{8\sqrt{2\pi}}{3} \frac{m_n}{m_d} R^2 n_n v_n,$$

where  $m_n$  and  $v_n$  are the neutral mass and thermal speed, respectively, and  $\eta$  is a numerical factor, which ranges from about  $\sim 1$  to 1.4 depending on whether the scattering is specular or diffuse and depending on the accommodation coefficient (see, e.g., Ref. [26]). In the following we use  $\eta = 1.4$  [26], so that  $v_d/\omega_{pd} \sim 0.3$ . The ion stream speed is given in Arp et al. [10] as being on the order of  $10^4$  cm/s, so that using  $V_{0i} = 1 \times 10^4$  cm/s yields  $V_{0i}/v_i \sim 0.4$ . However, this flow speed is estimated from simulation results but not measured, so we will also consider ion flow speeds comparable to  $v_i$  as indicated in Ref. [10]. If the ion streaming is due to an electric field,  $V_{0i}$  would result from balancing the electrostatic force with the neutral drag force on the ions, that is,  $V_{0i} = eE/m_iv_i$ . In this scenario, the electrons would stream in the opposite direction, with  $V_{0e}/v_e \sim (\sigma_{in}T_i/\sigma_{en}T_e)(V_{0i}/v_i)$ . Although we do not know what the dust coupling parameter  $\Gamma$  is for this system, we note that if  $T_d$  were about 0.35 eV (corresponding to a dust thermal speed of about 0.5 mm/s),  $\Gamma$  would be about 300.

Figure 2 shows the solution to (5) using the parameters in the previous paragraph. Note that the curves begin at values of  $k\lambda_{Di}$  where  $|\zeta_i| < 1$ , which roughly corresponds to the approximation in (4). The result for the weakly coupled fluid case with  $V_{0i}/v_i = 0.4$  is shown by the dot-dashed curves in Fig. 2, which is obtained by setting  $D_L(k) = 0$ . Note that the cold dust approximation used here should be appropriate when the dust acoustic speed  $c_{sd} \sim \lambda_{Di} \omega_{pd}$  is much greater than the dust thermal speed  $v_d$ . As can be seen, the weakly coupled fluid dust model does not predict growth for these parameters, although it is near marginal. If the dust collision frequency were somewhat smaller, due for example to a smaller value of  $\eta$ , or if ion collisional effects enhance the growth somewhat, or if the ion flow speed were larger, there might be growth even in the weakly coupled fluid dust case. For example, the dashed curves in Fig. 2 show the corresponding solution for the fluid dust model with the same parameters but with  $V_{0i}/v_i = 0.8$ , which does show DAI growth. On the other hand, if we assume the dust is strongly coupled in the liquid phase, with  $\kappa \sim 2$ and  $\Gamma \sim 300$ , the effect of strong coupling appears to predict substantial growth as shown by the solid curves in Fig. 2 even if the ion flow is  $V_{0i}/v_i = 0.6$ . As noted above,  $\Gamma \sim 300$ would correspond to a dust (kinetic) temperature of about



FIG. 2. (Color online) (a) Real frequency  $\omega_r$  and (b) imaginary part of frequency  $\gamma$  normalized to  $\omega_{pd}$  versus  $k\lambda_{Di}$  obtained by solving (5). The parameters are:  $m_i/m_p = 40$ ,  $T_e/T_i = 154$ ,  $n_i/n_e = 8$ ,  $\omega_{pd}/\omega_{pi} = 3 \times 10^{-5}$ , and  $v_d/\omega_{pd} = 0.3$ . The weakly coupled case, setting  $D_L(k) = 0$ , is shown for  $V_{0i}/v_i = 0.4$  (black, dash-dot curves) and  $V_{0i}/v_i = 0.8$  (blue, dashed curves). The strongly coupled case with  $\Gamma = 300$ ,  $\kappa = 2$ , is shown for  $V_{0i}/v_i = 0.6$  (red, solid curves).

0.35 eV. Unstable wavelengths appear to range from about 1.2 mm (corresponds to  $k\lambda_{Di} \sim 0.45$ ) at the longer end, while the observed wavelengths reported in Arp *et al.* [10] are 2 mm.

Next we consider the PK-4 microgravity experiments reported by Fortov et al. [12] (see also Khrapak et al. [11]). We consider a set of parameters with higher pressure and smaller dust grains than that considered above. These nominal parameters may be roughly representative of parameter ranges given in Ref. [12]: neon  $P \sim 50$  Pa, neon  $n_i \sim 3 \times 10^8$  cm<sup>-3</sup>,  $T_e \sim 7$  eV,  $T_i \sim 0.03$  eV,  $R \sim 0.6 \ \mu$ m, and  $n_d \sim 10^5 \ {\rm cm}^{-3}$ (giving  $a \sim 130 \,\mu$ m). With these values, the ion Debye length is  $\lambda_{Di} \sim 0.007$  cm, so that  $\kappa \sim 2$ . The dust charge is estimated to be  $Z_d \sim 1500$  from Fig. 7(a) in Khrapak *et al.* [11], which takes into account the reduction of the dust charge from the OML value due to ion-neutral collisions at higher pressure. Assuming the dust mass density is  $\sim 1.5 \text{ g/cm}^3$ , the dust mass is estimated as  $m_d \sim 8.2 \times 10^{11} m_p$ . Thus  $\omega_{pd} \sim 690$  and the the ratio  $\omega_{pd}/\omega_{pi} \sim 1.35 \times 10^{-4}$ . At this larger pressure, we estimate  $v_i/\omega_{pi} \sim 0.5$  and  $v_d/\omega_{pd} \sim 0.58$ . If we assume that an electric field of order 2 V/cm drives the ion streaming, we obtain an ion flow speed of about  $V_{0i}/v_i \sim 0.9$ . Again, we do not know the dust temperature, but if we assume  $T_d \sim$ 0.085 eV ( $v_d \sim 0.3$  cm/s) we have that  $\Gamma \sim 300$ . It should be noted, however, that in the ground-based PK-4 experiments reported in Ref. [11] the dust was streaming with speed  $\sim$ 5 cm/s. While this would lead to a Doppler shift in the observed wave frequency, fluctuations in the speed of individual dust grains could also lead to an increased effective dust temperature.



FIG. 3. (Color online) (a) Real frequency  $\omega_r$  and (b) imaginary part of frequency  $\gamma$  normalized to  $\omega_{pd}$  versus  $k\lambda_{Di}$  obtained by solving (5). The parameters are:  $m_i/m_p = 20$ ,  $T_e/T_i = 233$ ,  $n_i/n_e = 2$ ,  $\omega_{pd}/\omega_{pi} = 1.35 \times 10^{-4}$ ,  $v_d/\omega_{pd} = 0.58$ , and  $V_{0i}/v_i = 0.9$ . Weakly coupled case, setting  $D_L(k) = 0$  (black, dashed curves). Strongly coupled case, with  $\Gamma = 300$  and  $\kappa = 2$  (red, solid curves).

Figure 3 shows the solution to (5) using the parameters in the previous paragraph for the smaller dust grain case. Note that here again the curves begin at values of  $k\lambda_{Di}$  where  $|\zeta_i| <$ 1, very roughly corresponding to the approximation in (4). The weakly coupled fluid model with  $D_L(k) = 0$  (black, dashed curves) shows that no DAI growth is predicted. Growth of the DAI is shown in the strongly coupled case (red, solid curves) where  $\kappa = 2$  and  $\Gamma = 300$ , with the latter condition implying  $T_d \sim 0.085$  eV as mentioned above. The wavelength of the unstable mode at the longer end,  $k\lambda_{Di} \sim 0.6$  is about 0.8 mm.

## **IV. SUMMARY AND DISCUSSION**

We have considered the effects of strong coupling on the DAI in a dusty plasma in the strongly coupled liquid phase. The DAI is driven by ions streaming through the dust with speed less than the ion thermal speed. Due to strong coupling, growth of the DAI can be substantially enhanced due to a decrease in the wave frequency, particularly at k values where the frequency of the dust acoustic modes decreases and exhibits negative dispersion. We have also applied the predictions of the theory to parameters that may be representative of microgravity experiments where subthermal ion flows are in general possible. Although it appears that  $\kappa$  may be larger than unity in those experiments we do not know what  $\Gamma$  is primarily because we do not know what the dust temperature is. Assuming that the dust is in the strongly coupled liquid phase, though, it was found that strong coupling effects could lead to dust acoustic instability even when theory using a weakly coupled fluid model for the dust would predict stability due to dust collisional damping.



FIG. 4. Real frequency  $\omega_r$  and imaginary part of frequency  $\gamma$  normalized to  $\omega_{pd}$  versus  $k\lambda_{Di}$  obtained by solving (2), using (3) for all three charged species. The parameters for the dashed curves are:  $m_i/m_p = 40$ ,  $T_e/T_i = 154$ ,  $T_e/T_d = 0.1$ ,  $Z_d = 3500$ ,  $n_d/n_i = 2.5 \times 10^{-4}$ ,  $m_d/m_p = 1.4 \times 10^{14}$ ,  $v_d/\omega_{pd} = 0.3$ ,  $v_i/\omega_{pi} = 0.16$ ,  $v_e/\omega_{pi} = 53$ ,  $V_{0i}/v_i = 0.8$ , and  $V_{0e}/v_e = 0.05$ . The parameters for the dotted curves are:  $m_i/m_p = 20$ ,  $T_e/T_i = 233$ ,  $T_e/T_d = 0.58$ ,  $Z_d = 1500$ ,  $n_d/n_i = 3.3 \times 10^{-4}$ ,  $m_d/m_p = 8.2 \times 10^{11}$ ,  $v_d/\omega_{pd} = 0.58$ ,  $v_i/\omega_{pi} = 0.5$ ,  $v_e/\omega_{pi} = 85$ ,  $V_{0i}/v_i = 0.9$ , and  $V_{0e}/v_e = 0.06$ .

However, in order to better compare with the Vlasov theory for the weakly coupled phase, we should take dust thermal effects into account. In order for the dust to be in the weakly coupled phase, with  $\Gamma_d < 1$ , the dust kinetic temperature for micron size grains generally should be large. In this case, the cold dust approximation, which implies  $c_{sd} \gg v_d$ , may not be appropriate. Because  $c_{sd}/v_d \sim \sqrt{3\Gamma/\kappa}$ , the conditions  $c_{sd}/v_d \gg 1$  and  $\Gamma_d < 1$  may not be mutually compatible [27]. Therefore, when the dust is in the weakly coupled phase, we roughly model the dust susceptibility using the full kinetic

- N. N. Rao, P. K. Shukla, and M. Y. Yu, Planet. Space Sci. 38, 543 (1990).
- [2] V. E. Fortov, A. V. Ivlev, S. A. Khrapak, A. G. Khrapak, and G. E. Morfill, Phys. Rep. 421, 1 (2005).
- [3] Z. Donko, G. J. Kalman, and P. Hartmann, J. Phys.: Condens. Matter 20, 413101 (2008).
- [4] P. K. Shukla and B. Eliasson, Rev. Mod. Phys. 81, 25 (2009).
- [5] Complex and Dusty Plasmas, edited by V. E. Fortov and G. E. Morfill (CRC Press, Boca Raton, 2010).
- [6] M. Rosenberg and G. Kalman, Phys. Rev. E 56, 7166 (1997).
- [7] G. Kalman, M. Rosenberg, and H. E. DeWitt, Phys. Rev. Lett. 84, 6030 (2000).
- [8] M. Rosenberg, Planet. Space Sci. 41, 229 (1993).
- [9] M. Rosenberg, G. J. Kalman, and P. Hartmann, Contrib. Plasma Phys. 52, 70 (2012).
- [10] O. Arp, J. Goree, and A. Piel, Phys. Rev. E 85, 046409 (2012).
- [11] S. A. Khrapak, S. V. Ratynskaia, A. V. Zobnin, A. D. Usachev et al., Phys. Rev. E 72, 016406 (2005).
- [12] V. Fortov, G. Morfill, O. Petrov, M. Thoma *et al.*, Plasma Phys. Control. Fusion 47, B537 (2005).
- [13] M. Rosenberg and G. Kalman, in *Physics of Dusty Plasmas*, edited by M. Horanyi *et al.* (AIP, New York, 1998), pp. 135–141.
- [14] G. J. Kalman and M. Rosenberg, J. Phys. A: Math. Gen. 36, 5963 (2003).

expression (4) for all three charged particle species, in the dust rest frame with  $V_{0d} = 0$ . This was done for the parameters corresponding to Fig. 2, taking  $T_d = 40 \text{ eV} (\Gamma_d \sim 0.35)$ , and the solution of (2) for the weakly coupled gaseous phase is given by the dashed curves in Fig. 4. This was also done for the parameters corresponding to Fig. 3, taking  $T_d = 12 \text{ eV}$ (yielding  $\Gamma_d \sim 0.3$ ), and the corresponding solution of (2) is given by the dotted curves in Fig. 4. As can be seen, it appears that DAI growth is quenched in the weakly coupled gaseous phase for these parameter sets.

Although the present study does indicate trends of strong coupling effects on the DAI, there are a number of improvements that should be made in future work. This includes an investigation of the role of ion collisional effects along with better modeling of the ion susceptibility as  $V_{0i}$  approaches  $v_i$ . Another issue to be investigated is whether there is some correlation between the ions and the dust grains. As regards experiments to study these strong coupling effects on the DAI, the desirable parameters of microgravity experiments would include liquid phase systems with large  $\kappa$  in addition to measurable subthermal ion flow. For example,  $\Gamma = 725$  and  $\kappa = 3$  might be achieved in a system with  $n_i \sim 5 \times 10^8$  cm<sup>-3</sup>,  $T_i \sim 0.03$  eV,  $T_e \sim 4$  eV,  $a \sim 170 \,\mu$ m,  $R \sim 5 \,\mu$ m,  $Z_d \sim 10^4$ , and  $T_d \sim 1$  eV.

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- [15] M. Rosenberg and P. K. Shukla, Phys. Scripta 83, 015503 (2011).
- [16] G. Kalman and K. I. Golden, Phys. Rev. A 41, 5516 (1990).
- [17] K. I. Golden and G. J. Kalman, Phys. Plasmas 7, 14 (2000).
- [18] K. Miyamoto, *Plasma Physics for Nuclear Fusion* (MIT Press, Cambridge, 1989).
- [19] S. L. Ossakow, K. Papadopoulos, J. Orens, and T. Coffey, J. Geophys. Res. 80, 141 (1975).
- [20] A. F. Alexandrov, L. S. Bogdankevich, and A. A. Rukhadze, *Principles of Plasma Electrodynamics* (Springer-Verlag, Berlin, 1984).
- [21] B. D. Fried and S. D. Conte, *The Plasma Dispersion Function* (Academic Press, New York, 1961).
- [22] L. J. Hou, Z. L. Miskovic, A. Piel, and M. S. Murillo, Phys. Rev E 79, 046412 (2009).
- [23] Yan Feng, J. Goree, and Bin Liu, Phys. Rev. E 87, 013106 (2013).
- [24] P. K. Shukla, M. Torney, R. Bingham, and G. E. Morfill, Phys. Scripta 67, 350 (2003).
- [25] P. K. Shukla and A. A. Mamun, *Introduction to Dusty Plasma Physics* (Institute of Physics Publishing, Bristol, 2002).
- [26] S. A. Khrapak, A. V. Ivlev, and G. E. Morfill, Phys. Rev. E 70, 056405 (2004).
- [27] K. I. Golden and G. J. Kalman, arXiv:1204.6099 [physics.plasmph].