Phonon Spectrum in a Plasma Crystal

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The Fourier spectra of longitudinal and transverse waves corresponding to random particle motion were measured in a two-dimensional plasma crystal. The crystal was composed of negatively charged microspheres immersed in a plasma at a low gas pressure. The phonons were found to obey a dispersion relation that assumes a Yukawa interparticle potential. The crystal was in a nonthermal equilibrium, nevertheless phonon energies were almost equally distributed with respect to wave number over the entire first Brillouin zone.

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In a lattice at a finite temperature, particles move about in configuration space with random velocities and energies that have a Maxwellian distribution. The particle velocities, when Fourier transformed, correspond to a spectrum of waves, or phonons, with a wide range of frequency ω and wave number $\mathbf{k}$.

The energy of phonons corresponding to thermal motion is not distributed broadly over all $\mathbf{k}$-ω space; instead, it is concentrated at values permitted by a dispersion relation. In general, dispersion relations describe the preferred frequencies for normal modes, and they are valid for low amplitude, i.e., linear waves. The manner in which the energy is distributed among the wave numbers depends on the lattice's energetics; a lattice in thermal equilibrium will exhibit equipartition, so that each mode has the same energy, when averaged over time [1].

Here we report observations of phonons that we did not excite intentionally; they were present naturally, due to random particle motion. Our system was not in thermal equilibrium, but it nevertheless exhibited several characteristics typical of thermal equilibria. These include a Maxwellian distribution of velocities as measured in configuration space, and a phonon spectrum with wave energy distributed almost equally with respect to wave number.

Our system is a two-dimensional lattice formed by levitating polymer microspheres in a glow discharge plasma. The particles are immersed in a background of electrons and ions, which cause the particles to become negatively charged and interact through a screened Coulomb repulsion [2]. They are also immersed in a rarefied neutral gas, which induces a thermal Brownian motion and damps any organized particle motion. This kind of particle suspension is termed a dusty plasma [3]. Particles are levitated in the vertical direction, and trapped in the horizontal direction, by an electric field in the sheath region above a horizontal electrode. Our suspension was two dimensional because it had only enough particles to form a single layer, and the confining forces allowed measurable particle motion only in the horizontal plane. When their kinetic energy is limited by neutral gas damping, the particles tend to arrange themselves in an ordered structure [4–7], which is called a "plasma crystal" in analogy to a colloidal crystal. In our experiment, the particles formed a triangular lattice with hexagonal symmetry. The particle motion was observed directly by video microscopy, allowing us to determine the velocities of all particles in a sample area viewed by the camera.

Our lattice was a driven system. We measured the velocity distribution function, and found that it was Maxwellian, but with a temperature higher than the ambient neutral gas. The extra heating is presumably due to a fluctuating Coulomb force, but its exact mechanism is unknown. What is known with greater confidence is that this energy input is balanced by an energy loss due to neutral gas drag. Because the particles in the lattice simultaneously gain energy and lose energy by these external mechanisms, they form a driven system, which is a type of nonthermal equilibrium.

Waves in this kind of triangular lattice were observed previously in experiments by exciting them intentionally [8–10]. In those experiments, particle motion was manipulated by a powerful laser that pushed particles within an excitation region. As the laser beam was chopped on and off with a sinusoidal waveform, a wave propagated away from the excitation region, allowing a measurement of its wave number using a Fourier analysis method. Wave number was then plotted vs frequency, yielding an experimentally measured dispersion relation. The experimenters observed the two modes expected from theory [11], corresponding to longitudinal and transverse particle motion.

In this Letter, we report spectral measurements, but unlike the previous experiments we do not use external stimulation to excite the waves. Instead, we detect only the phonons that correspond to random particle motion.

The experimental apparatus is sketched in Fig. 1. An argon plasma was generated by rf glow discharge at a gas pressure of 18.6 mtorr, using at 13.56 MHz and 62 V peak to peak. The rf power was applied between a grounded vacuum chamber, which is not shown in Fig. 1, and a horizontal electrode. In the glow that formed above the horizontal electrode, the plasma had a density $\approx 3.5 \times 10^8$ cm^{-3} and an electron temperature ≈ 1 eV, as measured with a compensated Langmuir probe inserted into the main plasma region above the sheath.

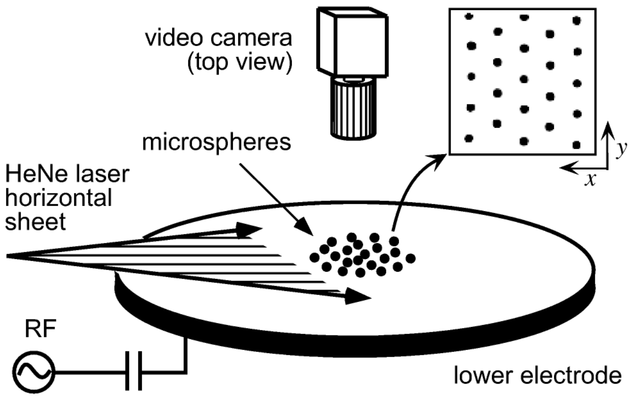

FIG. 1. Sketch of experimental setup. The inset is an image of part of the 2D lattice.

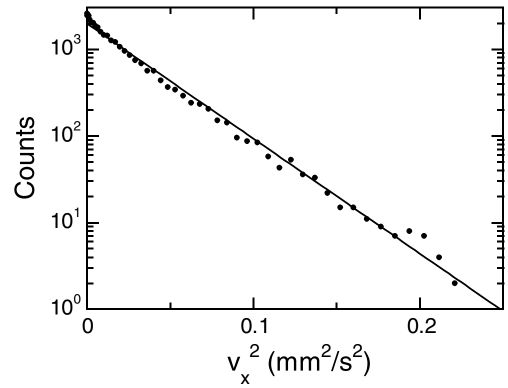

FIG. 2. Distribution function of particle velocities, as measured in configuration space. The line is a Maxwellian fit with $T = 0.046$ eV, which is hotter than the room temperature gas.

The particles that were dispersed into the plasma were melamine-formaldehyde microspheres with a 4.04 μm radius, a size dispersion with a standard deviation of $\pm 2.2\%$, and a mass density 1.51 g/cm^3, corresponding to a particle mass $m = 4.17 \times 10^{-13}$ kg/m^3. Approximately 5500 particles were introduced into the plasma, where they arranged themselves in a monolayer triangular crystal, approximately 7 cm in diameter. The interparticle spacing was $a = 0.90$ mm, as determined from the first peak of the pair correlation function, $g(\mathbf{r})$. Particles were illuminated with a low-power horizontal laser sheet, and they were imaged by a video camera with a field of view of 24×18 mm, including approximately 600 particles.

To analyze the particle motion, we digitized video images and identified each particle's x-y coordinates with subpixel resolution [10]. This was repeated for 128 frames at 15 frames per second. The particle velocity was calculated from the difference of particle positions in consecutive frames.

The particles had a velocity distribution that was nearly Maxwellian. This is shown in Fig. 2, which is a histogram of the squared particle velocity v_x^2, for only the x component of velocity. It exhibits an exponential decay over a wide range of v_x^2, which is fitted by a temperature $T = 0.046$ eV. We also calculated the kinetic energy from the mean square velocity, yielding $\langle m v_x^2 \rangle = 0.038$ eV. (The discrepancy in these two values reflects the uncertainty in the fit of Fig. 2.) For comparison, if our lattice were in thermal equilibrium with the gas, the particles would undergo Brownian motion with a temperature equal to the room temperature of the gas, 0.026 eV. This higher temperature indicates that our lattice was a driven system.

Using a Fourier transform method, the phonon spectrum is computed. From the components of the particle velocity $\mathbf{v}(\mathbf{r}, t)$, we calculated the wave amplitude $V_{\mathbf{k},\omega} = 2/TL \int_0^T \int_0^L v(\mathbf{r}, t) \exp[-i(\mathbf{k} \cdot \mathbf{r} - \omega t)]\, d\mathbf{r}\, dt$, where L and T are the length and the period over which a particle's motion is summed. This method is repeated for two kinds of polarization selections. First, we selected the angle θ of the wave propagation direction $\mathbf{k}$ with respect to a primitive translation lattice vector $\mathbf{a}$. Second, we selected either the longitudinal or the transverse mode by using only the component of $\mathbf{v}(\mathbf{r}, t)$ that is parallel or perpendicular to the wave propagation direction $\mathbf{k}$, respectively.

Our results for the natural phonon spectrum are shown in Figs. 3(a)–3(d). These are maps of the wave energy density in various subspaces of the full $\mathbf{k}$-ω space. Results are shown separately for two different propagation directions, $\theta = 0°$ and $90°$. The data cover the entire first Brillouin zone, i.e., $-2 < ka/\pi < 2$ for $\theta = 0°$, and $-2/\sqrt{3} < ka/\pi < 2/\sqrt{3}$ for $\theta = 90°$.

In Fig. 3, darker grays correspond to higher wave energy density. The grayness appears spotty due to statistics arising from the finite length of the data's time series. The energy density was computed for each frequency and wave number as $mV_{\mathbf{k},\omega}^2 / 2k\delta\theta\delta k\delta\omega k_B T$. Here, $\delta\theta$, δk, and $\delta\omega$ quantify the resolution by which we are able to distinguish waves with different values of θ, k, and ω, respectively, as will be described in another paper. In our experiment, the product $k\delta\theta\delta k\delta\omega$ was 0.0729 mm^{-2} s^{-1} for all the $\mathbf{k}$-ω space shown in Fig. 3.

Examining Figs. 3(a)–3(d), we note that the wave energy is concentrated on distinct curves in the various $\mathbf{k}$-ω subspaces; we identify these curves as the dispersion relations. The dispersion relations depend on the mode's propagation angle and on its polarization, longitudinal or transverse.

For comparison, we also measured the dispersion relations of the longitudinal and transverse modes using the laser-excitation method of Ref. [10], and this yielded the data points plotted in Figs. 3(i)–3(l). We find that these data for externally excited sinusoidal waves agree well with the spectra in Figs. 3(a)–3(d) for the natural phonons in the absence of any external stimulation. For the longitudinal wave in Fig. 3(a), it includes maxima in the frequency at $|ka/\pi| \approx 1$. For $|ka/\pi| > 1$ in Fig. 3(a), we note a downward slope, $(\omega/k)(d\omega/dk) < 0$, indicating a backward wave, with phase and group velocities that point

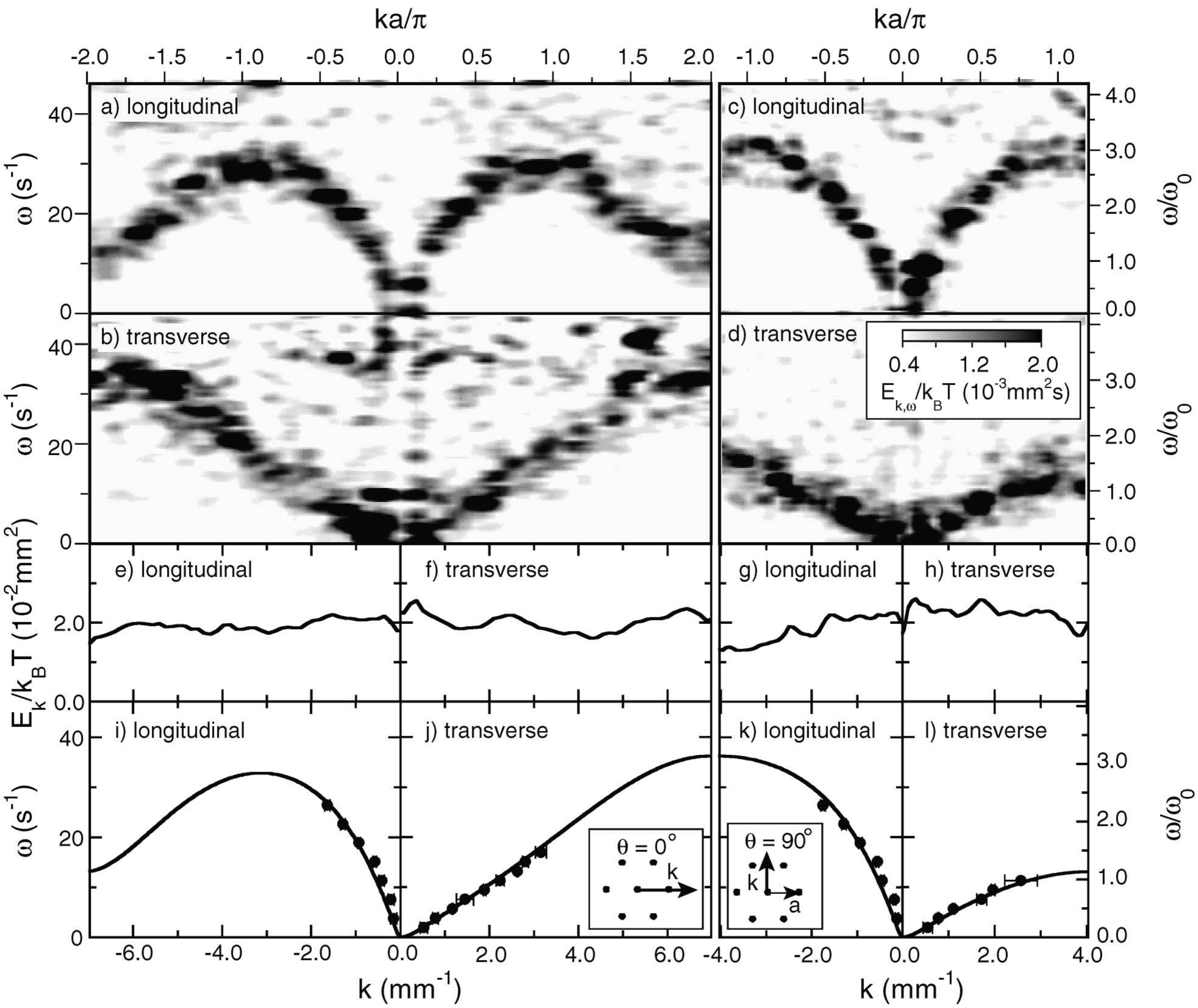

FIG. 3. Phonons in the first Brillouin zone. (a)–(d) Spectra of wave energy density in $\mathbf{k}$-ω space, in the absence of any intentional wave excitation. Darker grays correspond to higher wave energy. Energy is concentrated along a curve corresponding to a dispersion relation. (e)–(h) Spectra integrated over ω, showing that wave energy is distributed nearly uniformly with respect to wave number. (i)–(l) Theoretical dispersion relation (curves) shown fitted to the experimental dispersion relation for waves excited intentionally using a laser (circles). The angle θ between $\mathbf{k}$ and $\mathbf{a}$ is 0° in the left panels, 90° in the right panels.

in opposite directions. In contrast, the transverse wave in Fig. 3(b) is essentially dispersionless, i.e., $\omega \propto k$, over nearly the entire first Brillouin zone.

The waves due to random motion in Figs. 3(a)–3(d) were detected for all allowed wave numbers. In contrast, the laser was unable to excite the largest wave numbers in Figs. 3(i)–3(l), which we attribute partly to the laser beam's finite size. The wave numbers that the laser can excite efficiently are those that are comparable to the reciprocal of the laser beam's width. Laser-excited waves also experience heavy damping at frequencies where the group velocity is zero, $d\omega/dk = 0$ [12].

Our spectra agree well with the theoretical dispersion relations shown as solid curves in Figs. 3(i)–3(l). We use the theory derived by Wang *et al.* [12] from the equation of motion for a 2D triangular lattice with a Yukawa potential. This agreement indicates that the Yukawa potential is a good approximation for the particle interaction in our 2D crystal. We used two adjustable parameters, $\kappa \equiv a/\lambda_D$ and particle charge Q, to fit theory to experiment in Fig. 3, yielding $\kappa = 1.2$, $Q/e = 13\,000$. The damping rate was assumed to be 3.7 s^{-1}, according to the particle radius and the gas pressure. These results, together with our measurements of a and T, yield the Coulomb coupling parameter $\Gamma \equiv Q^2/4\pi\varepsilon_0 a k_B T = 7500$, and $\omega_0 \equiv (Q^2/4\pi\varepsilon_0 m a^3)^{1/2} = 11.6$ s^{-1}. Finally, we note that the theoretical dispersion relation is valid for linear or small amplitude waves, and is able to show agreement with our spectra in Figs. 3(a)–3(d) because our phonons have a small amplitude.

We now turn our attention to quantitative measurements of the wave energy in the spectra of Figs. 3(a)–3(d). The results presented below reveal that our driven system seems to have properties of a system in thermal equilibrium.

We find empirically that the wave energy was distributed equally over all values of **k** in the first Brillouin zone. This conclusion is drawn from Figs. 3(e)–3(h), which are **k** spectra of the phonons, computed by integrating the spectra in Figs. 3(a)–3(d) over frequency to yield a wave energy $E_{\mathbf{k}}$ as a function of **k**. Because all reciprocal lattice vectors are placed at regular intervals in **k** space, this result indicates that the thermal energy of the plasma crystal is distributed nearly equally into each possible mode. Such a result would be expected if the lattice were in thermal equilibrium, which would obey equipartition, although our lattice is a driven system.

We can compare the total energy E in our wave spectrum with the kinetic energy K as measured in configuration space. For 2D systems, $E = N \sum_{\mathrm{polarization}}^{2} \int_{\mathbf{k}} \int_{\omega} E_{\mathbf{k},\omega}\, d\omega\, d\mathbf{k}$, and $K = Nk_BT$ using the temperature determined from particle velocities in configuration space. Here, N is the total number of particles, and "polarization" refers to the two modes, longitudinal and transverse. The area of integration in 2D **k** space is the first Brillouin zone, and the range of integration on the ω axis is zero to infinity. For our plasma crystal, we use our conclusion from Figs. 3(e)–3(h), that $E_{\mathbf{k}}$ is constant over the entire area of the first Brillouin zone in a 2D reciprocal lattice. Repeating the experiment at various conditions, with a shielding parameter κ in the range 0.78–1.32 and gas pressure in the range 18–33 mtorr, we found that E/K was in the range 1.5–3.1. (For the experimental conditions corresponding to Figs. 2 and 3, $E/K \approx 2.2$.) For comparison, we note that, for systems in thermal equilibrium, which obey the Virial theorem, $P = K$ and $E/K = 2$.

Finally, we present a frequency spectrum of particle velocities in Fig. 4. This spectrum was computed from particle velocities parallel to the primitive lattice vector, using $V_\omega = \frac{2}{T} \int_0^T v_y(t) \exp(-i\omega t)\, dt$, which is insensitive to wave number and polarization. We note that the frequency spectrum in Fig. 4 has two peaks, at $\omega/\omega_0 \approx 1.0$ and 2.5. The reason for these peaks becomes clear by noticing that they correspond to features in Figs. 3(a) and 3(d), where $d\omega/dk \approx 0$. This happens at $\omega/\omega_0 \approx 1.0$ for both the longitudinal and the transverse modes and at $\omega/\omega_0 \approx 2.5$ for only the longitudinal mode. Because energy is distributed nearly equally with respect to wave number, there is more energy at frequencies for which there are many wave numbers, $d\omega/dk \approx 0$.

In summary, we measured a phonon spectrum due to random particle motion. It obeys the dispersion relations for longitudinal and transverse waves over the entire first Brillouin zone. We have shown three results for our driven system that seem to match what is expected for a thermal equilibrium. First, the particle velocities as measured in configuration space have a Maxwellian velocity distribution. Second, the phonons, as measured in Fourier space, have energies that are almost equally distributed with respect to wave number. Third, the wave energy measured in Fourier space is approximately twice as large as the particle kinetic energy measured in configuration space.

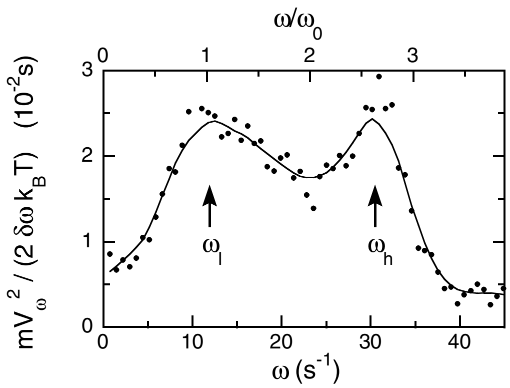

FIG. 4. Frequency spectrum in the absence of any intentional wave excitation. Modes of all wave numbers and polarizations are included. Experimental data are shown along with a smoothed curve. The two peaks correspond to the features in Fig. 3, where $d\omega/dk \approx 0$.

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