Ion Trapping by a Charged Dust Grain in a Plasma

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Motivated by space and laboratory applications, a theory is presented for ions trapped by a charged dust grain in a plasma. The grain's attractive Debye sphere confines ions after a collision. They shield the grain's considerable electric charge from external fields. Their number N_{trap} is determined by a balance of the capture and loss rates. At steady state, N_{trap} is independent of the collisional mean free path and increases with the plasma ion density. Because of the density dependence, trapping is significant in laboratory plasmas but not in comparatively less dense space plasmas.

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Dusty plasmas are low-temperature ionized gases that contain particulates in the size range 10 nm to 100 μm. These dust grains collect ions and electrons, and acquire an electric charge Q_D, which can be thousands of electron charges [1,2]. Charged grains can modify and even dominate wave propagation [2,3], wave scattering [4], ionization balance [5], shock propagation [5], and gradient and velocity-space-driven instabilities [6,7]. In space and astrophysics [8], theories have been developed to explain observations in planetary rings [2], interstellar clouds [5], Earth's noctilucent clouds [9], and spacecraft-ionosphere interactions [10]. Laboratory dusty plasma experiments have been performed only recently; they reveal that charged particulates can be confined for hours in a discharge [11-13]. This confinement leads to a serious contamination problem in the plasma-aided manufacturing of semiconductors [11,12], while it offers novel research opportunities for basic plasma physics [14].

The starting point for every theory of dusty plasmas is a charging model. Now I summarize a standard model. Here and throughout this Letter I treat only the most common case: $a \ll \lambda_D \ll \lambda_{mfp}$, where a is the grain radius, λ_D the Debye length, and λ_{mfp} the collisional mean free path. The charge Q_D is related to the grain's floating potential ϕ_f by $Q_D = C\phi_f$, where $C = 4\pi\varepsilon_0 a$ is the grain's capacitance. A grain collects ion and electron currents from the plasma [1,2], and these currents J_σ depend on ϕ_f. They add up to determine the charge, $dQ_D/dt = \sum J_\sigma(\phi_f)$.

Setting this sum to zero yields the steady-state floating potential ϕ_f and charge Q_D. Because of the low electron mass, ϕ_f and Q_D are always be negative, unless the grain emits electrons. A sphere in a thermalized hydrogen plasma float to $\phi_f = -2.51kT/e$, where T is the ion and electron temperature [15]. This yields a charge $Q_D = -1.7\times 10^4 e$, for example, if $kT = 1$ eV and $a = 10$ μm. The actual potential and charge are somewhat less negative than this, if ions drift with respect to the grain faster than the ion thermal speed [1].

This standard charging model ignores particles that become trapped in the Debye sphere surrounding a charged grain. Because the grain has a large negative charge, it can easily hold one or more positive ions in confined orbits, as shown in Fig. 1. Trapped ions are important because they, unlike untrapped ions, shield the grain's charge from external electromagnetic fields. This is similar to the way that electrons bound in an atom cancel the charge of the nucleus within. Electromagnetic forces on the grain are thereby reduced, and other forces, such as gravity and gas drag, become more significant. The larger the number of trapped ions, N_{trap}, the more effective the shielding.

The physics of trapped ions was presented earlier in theories for probes [16] and spacecraft [17], but never developed to a useful end. Consequently theorists have neglected trapping in computing the forces acting on dust in a plasma, without knowing the validity of this assumption. To start, it would help to have a criterion to answer the question: When is it valid to neglect trapping? Such a criterion is $N_{trap} < 1$. This requires a method of computing N_{trap}, which is developed below.

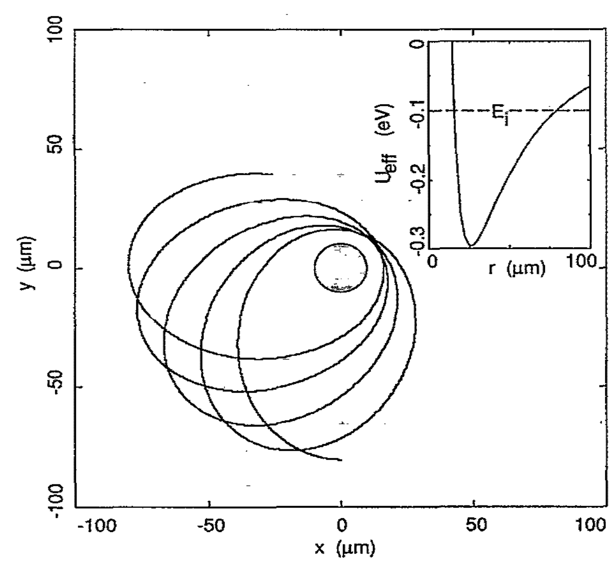

FIG. 1. Trapped hydrogen ion orbiting around a negatively charged dust grain. Parameters are $E_i = -0.1$ eV, $\phi_f = -2.5$ V, and others listed for Fig. 2. $U_{eff}(r)$ is shown in the inset.

To compute $N_{\rm trap}$, consider a positive ion, with charge q_i, mass m_i, and angular momentum M, moving toward a negatively charged grain. The ion's total energy is $E_i=m_i\dot{r}^2/2+U_{\rm eff}(r)$, where $U_{\rm eff}(r)=q_i\phi(r)+M^2/2m_ir^2$. An ion is trapped if it is in a potential well, i.e., if it satisfies three requirements: its $U_{\rm eff}$ has a minimum, E_i is less than the maximum of $U_{\rm eff}$ found at nonzero r, and the orbit does not strike the grain. An untrapped ion can become trapped only if it undergoes a collision, scattering into a favorable direction and losing enough energy.

The rate of ion capture is

$$\dot{N}_{\rm cap}=\Gamma_i\sigma_{\rm cap}\,, \tag{1}$$

where $\Gamma_i=n_iv_i$ is the incident ion flux. Here $\sigma_{\rm cap}$ is a capture cross section

$$\sigma_{\rm cap}=\int_0^\infty 2\pi p\,dp\,P_{\rm cap}(p)\,, \tag{2}$$

and $P_{\rm cap}(p)$ is the probability for an incoming ion at impact parameter p to become captured due to a collision. Note that $P_{\rm cap}$ and $\sigma_{\rm cap}$ are $\propto 1/\lambda_{\rm mfp}$.

What sort of collisions must be considered? Only those that result in slower ions will lead to capture. For neutral atoms and ions that are alike, charge-exchange and elastic collisions have comparable cross sections. But charge exchange yields the most ion energy loss, provided that $kT_{\rm gas}\ll E_i$.

The loss process is also collisional. Eventually a collision will scatter an ion enough that it becomes untrapped, either escaping to infinity or striking the grain. The loss rate is

$$\dot{N}_{\rm loss}=N_{\rm trap}/\tau_{\rm conf}\,, \tag{3}$$

where $\tau_{\rm conf}$ is a confinement time. To make an estimate valid for isotropic scattering, it is useful to know that only one or two collisions are needed to detrap an ion; thus $\tau_{\rm conf}\approx\lambda_{\rm mfp}/v_{\rm trap}$, where $v_{\rm trap}$ is a typical speed of trapped ions. A more precise value requires numerical evaluation, as done below.

A steady state is achieved when the capture and loss rates balance, $dN_{\rm trap}/dt=\dot{N}_{\rm cap}-\dot{N}_{\rm loss}=0$. This and Eqs. (1) and (3) yield

$$N_{\rm trap}=\Gamma_i\sigma_{\rm cap}\tau_{\rm conf}\,. \tag{4}$$

This is a mean value for the number of trapped ions; the instantaneous number fluctuates in time.

Two conclusions about the physics of trapping can be made from Eq. (4). First, the number $N_{\rm trap}$ of collisionally trapped ions is independent of the collisional mean free path, because the $\lambda_{\rm mfp}$ dependences of $\sigma_{\rm cap}$ and $\tau_{\rm conf}$ in Eq. (4) cancel. Provided that charge exchange is the dominant collision process, it does not matter how low the gas density is. Second, the dependence of Γ_i means that the number of trapped ions is $\propto n_i$. A grain will trap more ions if it is in a denser plasma.

I have implemented the physical model, described above, as a single-particle Monte Carlo code. The simulation has two stages. First, an incoming ion with $E_i>0$ is directed toward the dust grain, at an impact parameter p. The equation of motion, $\ddot{\mathbf{x}}=-(q_i/m_i)\nabla\phi$, is integrated in three dimensions using a fourth-order Runge-Kutta method. To determine if a collision took place in a time step Δt, the probability $1-\exp(-\Delta t|\mathbf{v}|/\lambda_{\rm mfp})$ is compared to a random number. The ion orbit is followed until it either undergoes a collision or escapes from the simulation volume, a sphere of radius $5\lambda_D$. At a charge-exchange collision site, a new ion is born, and its initial velocity is chosen randomly from the gas thermal distribution. It is evaluated against the three requirements to see if its orbit is trapped. This is repeated for 200 to 1000 test particles, all with the same p. The capture probability $P_{\rm cap}(p)$ needed for Eq. (2) is then calculated; it is the fraction of incoming ions that yielded a trapped ion. This first stage is repeated for forty impact parameters. Computing the integral in Eq. (2) then yields $\sigma_{\rm cap}$.

In the second stage, the new trapped ion orbits are followed one at a time by integrating the equation of motion. They are followed until, due to collisions, they either strike the grain or escape the simulation space. A histogram of the trapped orbit lifetime is prepared, and it is fitted by a decaying exponential, $N_{\rm trap}=N_0\exp(-t/\tau_{\rm conf})$, based on Eq. (3). This procedure yields the confinement time $\tau_{\rm conf}$.

I make the following simplifying assumptions. First, only charge-exchange collisions are included, and second, their cross section is constant (over a decade range of energy). Third, the incoming ions are monoenergetic; this is appropriate where $E_i\gg T_i$. Fourth, the neutral gas does not drift with respect to the grain. Fifth, the grain is spherical. Finally, the electric potential is

$$\phi(r)=\frac{Q_D}{4\pi\varepsilon_0 r}\exp\left(\frac{a-r}{\lambda_D}\right), \tag{5}$$

which neglects any trapped orbits or wake effects [1,18]. This prescribed electric potential does not evolve in time as it would in a self-consistent particle simulation. Equation (5) is valid only if (a) the number of trapped ions is small, $N_{\rm trap}\ll Q_D/e$, which may be verified from the results; (b) the Debye spheres of two grains do not overlap [19,20]; (c) λ_D is much smaller than the ion gyroradius, so that any magnetic field may be ignored; and (d) there is no external potential drop across the Debye sphere, as found, for example, in the electrode sheaths of a high-voltage laboratory discharge. In the latter case, I speculate that a grain with a cloud of trapped ions will lose those ions when it approaches a sheath; they will probably be detrapped when their orbits cross into the sheath.

The simulation was performed for hydrogen, with the parameters $a=10\ \mu$m, $\lambda_D=100\ \mu$m, $\lambda_{\rm mfp}=3$ mm, $E_i=1$ eV, and $T_{\rm gas}=406$ K ($kT_{\rm gas}=0.035$ eV). These parameters satisfy the conditions $a\ll\lambda_D\ll\lambda_{\rm mfp}$ and $kT_{\rm gas}\ll E_i$. Results are shown in Figs. 2 and 3. The capture cross

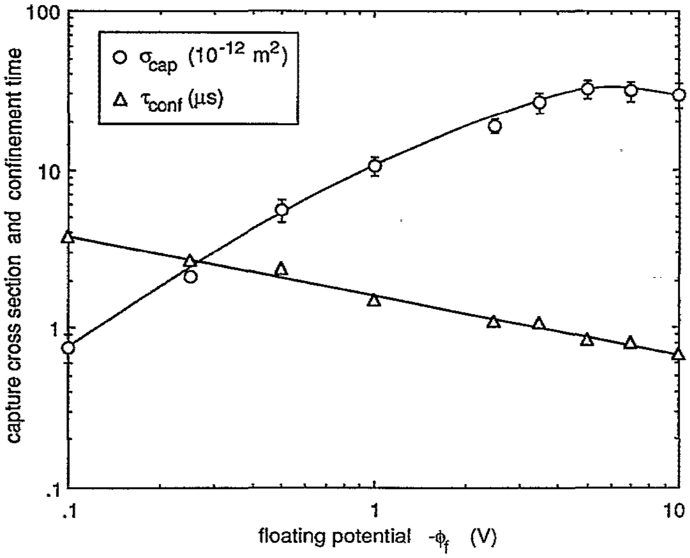

FIG. 2. Monte Carlo simulation results for captured hydrogen ions, born by charge exchange. The capture cross section σ_{cap} is computed from Eq. (2). Newly captured ions were followed until they were detrapped due to further collisions, yielding the confinement time τ_{conf}. Parameters are $a=10$ μm, $\lambda_D=100$ μm, $\lambda_{mfp}=3$ mm, $E_i=1$ eV. Error bars arise from counting statistics.

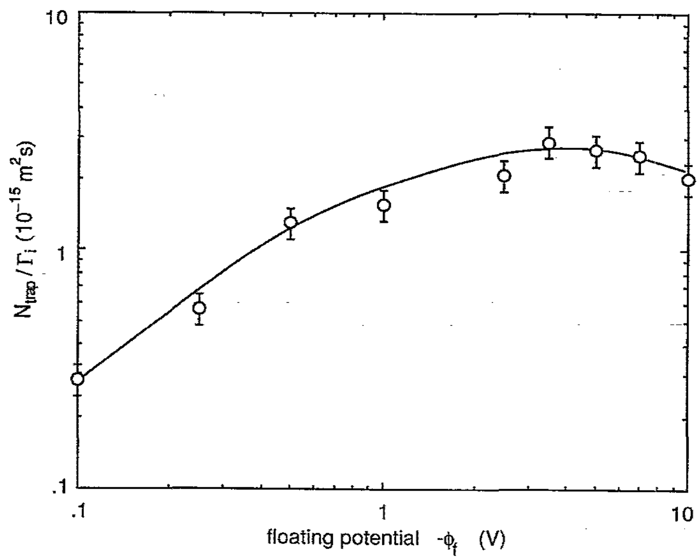

FIG. 3. Number of trapped ions N_{trap} per unit of incoming ion flux, $\Gamma_i=n_i v_i$. These results from Eq. (4) are valid if $N_{trap} \ll Q_D/e$. Unlike Fig. 2, these data are independent of λ_{mfp}.

section σ_{cap} in Fig. 2 increases with the floating potential, since ϕ_f is attractive. The confinement time τ_{conf} in Fig. 2 diminishes with ϕ_f, because $\tau_{conf} \approx \lambda_{mfp}/v_{trap}$, and ions trapped in a shallower potential have a lower velocity v_{trap}. (This result is due in part to assuming a collision cross section that is independent of energy.) Figure 3 shows the final result, N_{trap}/Γ_i as a function of floating potential, which is useful in two ways.

First, Fig. 3 can be used to compute N_{trap}, provided that the ion flux is low enough that $N_{trap} \ll Q_D/e$. For higher fluxes this will not be satisfied; N_{trap} will no longer increase linearly with Γ_i, but rather saturate at Q_D/e. A model with a self-consistent electric potential would be required to treat the saturation regime.

Second, Fig. 3 allows an evaluation of the criterion $N_{trap} < 1$, for ion trapping to be negligible. For example, Fig. 3 shows that $N_{trap}/\Gamma_i = 10^{-15}$ m^{2}s for $\phi_f = -0.5$ V. If ions are streaming past the grain at $v_i = 10^3$ m/s, then N_{trap} will be unity for $n_i = 10^{12}$ m^{-3}. In space plasmas $n_i \ll 10^{12}$ m^{-3}, while in laboratory discharges $n_i \gg 10^{12}$ m^{-3}. This shows that ion trapping is negligible in space physics problems, a finding that is a confidence builder for extant space dusty plasma models that neglect ion trapping. But trapping in laboratory plasmas is so strong that it is often in the saturation regime. Previous calculations [14,21] of electric forces acting on grains in laboratory plasmas have ignored trapping; they are probably invalid except for regions within a Debye length of a strong sheath.

The results for N_{trap} presented in Fig. 3 were computed for particular values of several parameters: a, λ_D, λ_{mfp}, E_i, T_{gas}, and ion mass. The results can be extended to other parameter values by using the following physics. The results are independent of λ_{mfp} provided that $\lambda_{mfp} \gg \lambda_D$. Scaling all the relevant energies (E_i, kT_{gas}, and $e\phi_f$) by the same factor will leave N_{trap} unaffected. The results should not depend strongly on the incoming ion energy E_i, since it affects only the spatial distribution of collision sites, which is always fairly uniform.

In summary, negatively charged dust grains in a plasma can trap positive ions in confined orbits, shielding the grain from external electromagnetic fields. The number of trapped ions is determined by a balance between collisional trapping and detrapping. A method of computing N_{trap} was developed here, using a non-self-consistent electric potential, valid if N_{trap} is small compared to the number of electron charges on the grain. A theory with a self-consistent electric potential still needs to be developed to compute N_{trap} when it is large. I find that ion trapping is very significant in laboratory plasmas, but negligible in most space plasmas.

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