Particle Interaction Measurements in a Coulomb Crystal Using Caged-Particle Motion

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A technique for characterizing the particle interaction potential of a Coulomb crystal is developed. The mean-square displacement (MSD) is measured, showing both caged- and superdiffusive-particle motions. By subtracting the center of mass of neighboring particles in computing MSD, only short-wavelength particle motions are retained. This yields the lattice Einstein frequency, which contains information about the interparticle forces and potentials. Video measurements of particle motions in a complex (dusty) plasma are used to demonstrate the technique.

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Coulomb crystals have been studied for many years and can be thought of as models for natural crystals. A Coulomb crystal is a system of mutually repulsive particles that has self-organized into a lattice structure under the influence of an external confinement. It exhibits many of the same properties as ordinary crystals, such as the development of topological defects and phase transitions during heating. Thus, a detailed study of the properties of a Coulomb crystal yields useful information about ordinary crystals.

Here, we describe a technique for characterizing the particle interaction potential in a Coulomb crystal. The methodology is generally applicable since it requires only measurement of the mean-square displacement (MSD) of the particles in the lattice, as a function of time. Particles in a crystal, or a highly ordered or freezing liquid, cannot diffuse freely but are trapped at short times by the "cage" formed by the neighboring particles.

We filter out the particle motion associated with long-wavelength phonons by computing the MSD of a particle relative to the center of mass of the neighboring particles. What remains are short-wavelength phonons which yield information about the interparticle forces and potentials through the Einstein frequency ω_E.

We demonstrate this technique using a two-dimensional (2D) experimental model system called a complex (or dusty) plasma. The complex plasma, so called in analogy with complex fluids, consists of a suspension of highly charged particles in a background plasma of ions and electrons and confined by external electric fields. The particle motions are damped by collisions with the neutral background gas, and phase transitions are frequently driven by changing the neutral gas pressure [1,2]. Complex plasmas are similar to aqueous colloidal suspensions [3] but have damping rates and volume fractions which are smaller by a factor of up to 10^5. The ambient plasma plays several roles: it sustains a negative charge on the particles, it provides a sort of Debye shielding in the vicinity of the particles, and it provides an inward long-range electric force $\boldsymbol{F}_{\text{ext}}$ that confines the mutually repulsive particles in a stable suspension. The suspension is characterized by direct measurements of the particle locations which yield structural, such as topological defect statistics, as well as dynamical information.

In the time domain, the dynamical measurements we report here are MSD(t) and the mean-square velocity $\langle v^2 \rangle$. In previous experiments, MSD(t) has revealed diffusive and superdiffusive motion at long times [4]; our data show this as well, but we add analysis of short-time caged (oscillatory) motions, from which we also derive ω_E. Note that it is possible to measure the MSD not only from direct measurements of particle positions, as we have done, but also, e.g., using dynamic light scattering in complex fluids [5]. In the frequency domain, dynamical measurements can also be made [6]; however, the spectra do not yield ω_E in a straightforward way.

Information about the interparticle potential is embedded in the MSD through the particles' caged motions. The equation of motion for a caged particle can be modeled with a single-particle Langevin equation:

$$\frac{d^2x}{dt^2} = -\omega_E^2 x - \nu \frac{dx}{dt} + \frac{1}{m}\xi(t). \tag{1}$$

Here, $x(t)$ is the coordinate of a single particle, m is the particle mass, $m\nu\, dx/dt$ is the drag force due to the neutral gas, $m\omega_E^2 x$ is the springlike force due to the cage of the neighboring particles, and $\xi(t)$ is a random, fluctuating force that heats the particle [7]. The lattice Einstein frequency ω_E is obtained from a Taylor series expansion of the net force on a particle due to the particles around it. In turn, the net force depends on the interparticle potential and lattice configuration.

Equation (1) is the equation of motion for a driven, damped harmonic oscillator, and it can be solved for MSD(t) $\equiv \langle \boldsymbol{x}^2/a^2 \rangle$ yielding

$$\text{MSD}(t) = \frac{\langle v^2 \rangle}{a^2\omega_E^2}\left\{1 - \exp(-\nu t/2)\left[\cosh(\nu_0 t) + \frac{\nu}{2\nu_0}\sinh(\nu_0 t)\right]\right\}, \tag{2}$$

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