Cutoff Wave Number for Shear Waves in a Two-Dimensional Yukawa System (Dusty Plasma)

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The cutoff wave number for shear waves in a liquid-state strongly coupled plasma was measured experimentally. The phonon spectra of random particle motion were measured at various temperatures in a monolayer dusty plasma, where microspheres interact with a Yukawa potential. In the liquid state of this particle suspension, shear waves were detected only for wavelengths smaller than 20 to 40 Wigner-Seitz radii, depending on the Coulomb coupling parameter. The temperature of the suspension was controlled using a laser-heating method.

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The waves in molecular solids or liquids can rarely be studied experimentally at an atomistic level. The reasons include small distances between the atoms or molecules in regular matter, high characteristic frequencies, and lack of experimental techniques of visualizing the motion of individual atoms or molecules. A suitable model system to experimentally study waves at an atomistic level is a dusty plasma. A dusty plasma is a suspension of highly charged micron-size particles in a plasma. When these particles are confined, their mutual repulsion causes them to self-organize in a structure that can have a crystalline or liquid order. In a dusty plasma the interparticle distance can be of the order of 1 mm, characteristic frequency of the order of 10 s^{-1}, and the speed of sound of the order of 10 mm/s. This ordered structure is vastly softer than molecular materials and even colloidal crystals, so that it can be manipulated easily using even the very weak force of radiation pressure applied by a laser beam. These unique characteristics, plus a very helpful possibility of direct imaging, make it possible to study complex phenomena like wave propagation, phase transitions, and transport phenomena, all at an atomistic level.

Waves in dusty plasmas have recently attracted much attention. The theory of different modes has been developed for two-dimensional (2D) [1–4] and three-dimensional (3D) [3,5–8] dusty plasmas in solid and liquid states. For strongly coupled dusty plasmas in the solid state, the linear modes for compressional and shear motion [9,10] have been extensively studied experimentally. For dusty plasmas in a liquid state, however, only a few experimental studies of wave modes have been reported so far [11–13]. For 2D dusty plasmas, one reason for this scarcity is that 2D dusty plasmas tend to crystallize under normal experimental conditions.

Liquids generally support compressional (longitudinal) waves. However, shear (transverse) waves can only propagate if their wavelength is as short as a few molecular spacings. Thus, the dispersion relation of the shear waves in a liquid has a cutoff wave number. This cutoff is well-known for molecular liquids [14,15], and it has been predicted theoretically for strongly coupled plasmas as well [4,7]. To the best of our knowledge, however, the cutoff wave number has never been observed experimentally in strongly coupled plasmas. Unlike the scattering methods used to detect phonons in molecular liquids, here we will use direct observation of random particle motion at an atomistic level, and by analyzing this random motion we will measure a phonon spectrum that reveals the wave number cutoff.

Using strongly coupled dusty plasmas, experiments to observe shear waves in a liquid state have only recently begun. In Ref. [12], a liquid state of a 2D dusty plasma was achieved by placing a varying amount of perturbing particles in a lower incomplete layer. Compressional and shear waves polarized in the plane of the particle suspension were studied in the solid and liquid states. The dispersion relation of the shear mode did not resolve a cutoff. In Ref. [11], the shear waves with a vertical polarization were observed in a 3D liquid-state dusty plasma. Their dispersion relation was measured and found to agree with a viscoelastic theory. Because of the lack of experimental data points near $\omega = 0$, no conclusion can be made on the existence of the cutoff wave number.

In this Letter, we use our laser-heating method [16] to melt a 2D dusty plasma crystal and to control the temperature of the resulting liquid. This allows us to study waves in a 2D dusty plasma in solid and liquid states, at various temperatures. The waves correspond to random particle motion at a given temperature. Unlike our experiment in Ref. [17], where we launched sinusoidal waves, here we did not use any additional external means to excite these waves.

The experimental setup was as in Refs. [16,17], using similar experimental parameters. Argon plasma was produced using a capacitively-coupled rf discharge. The discharge was sustained by 35 W of rf power at 13.56 MHz. To reduce the gas friction, Ar was used at a relatively low pressure of 5 mTorr. In this case, the neutral-gas damping rate ν is accurately modeled [18] by the Epstein expression with a leading coefficient of 1.26. In our experiment, $\nu = 0.87\ \mathrm{s}^{-1}$, so that particle motion on the time scales studied here was not overdamped.

A monolayer of microspheres was suspended in the plasma. The particles were highly charged as they absorbed more electrons than ions and they were levitated against gravity in the sheath above the lower rf electrode. The particles had a diameter of $8.09 \pm 0.18\ \mu\mathrm{m}$ [18] and a mass $m = 4.2 \times 10^{-13}$ kg. The suspension included $\approx$ 6700 particles, had a diameter of 50–60 mm, and rotated slowly in the horizontal plane.

The interparticle potential for particles arranged in a single plane, like ours, was experimentally shown [19] to be nearly Yukawa: $U(r) = Q(4\pi\epsilon_0 r)^{-1}\exp(-r/\lambda_D)$, where Q is the particle charge and λ_D is the screening length. The particle suspension is characterized by screening parameter $\kappa = a/\lambda_D$ and coupling parameter $\Gamma = Q^2/4\pi\epsilon_0 akT$, where T is the particle kinetic temperature. Because of the laser-beam configuration, the temperature was anisotropic with $T_x = (1.2\text{–}1.5)T_y$ [16]; here we compute temperatures as $T = T_y = m\langle(v_y - \bar{v}_y)^2\rangle/k_B$, where v_y is the y component of the particle velocity (transverse to the direction of the laser beam). We chose $T = T_y$ because in the shear waves that we study here the particle displacement was in the y direction. For liquids, the characteristic length is the 2D Wigner-Seitz radius $a = (\pi n)^{-1/2}$, where n is the areal number density [4]; it is related to the lattice constant b for a perfect triangular lattice by $a = (\sqrt{3}/2\pi)^{1/2}b$. In our experiment, $a = 0.325$ mm. We used the pulse technique of Ref. [20], making use of a theoretical wave dispersion relation [3], to measure $\kappa = 0.43 \pm 0.06$ and $Q = -12\,800 \pm 1200e$.

The particles were imaged through the top window by a video camera. We digitized movies of 2048 frames at 29.97 frames per second. The $22.7 \times 17.0\ \mathrm{mm}^2$ field of view included $\approx$ 1100 particles. The particle coordinates x, y and velocities v_x, v_y were then calculated with subpixel resolution [21] for each particle in each frame. Additionally, using a side-view camera we verified that no out-of-plane buckling occurred; we can therefore state that all experimental results reported here are for a single horizontal layer of particles.

At our experimental conditions, the particle suspension self-organized in a highly ordered triangular lattice. We used our laser-heating method that we explained in detail in Ref. [16] to melt the lattice and to control the temperature of the resulting liquid dusty plasma.

Two laser beams with a wavelength of 532 nm were pointed toward the suspension from opposite sides at a grazing angle. Particles were pushed by the radiation pressure force. The laser beams were moved about using scanning mirrors, so that the beam footprints drew Lissajous figures on the suspension. The two Lissajous frequencies were $f_x = 9$ Hz and $f_y = 14.5623$ Hz. The region covered by the Lissajous figures was an elongated rectangle covering the entire suspension in the x direction and its 12.4 mm wide central part in the y direction. This scheme provided brief intense random kicks to the particles in the heated stripe. As a result of these kicks and subsequent collisions, the particle kinetic energy increased, and the suspension had properties of a system in thermal equilibrium, as we discussed in detail in Ref. [16].

By varying the output power of our manipulation laser, we were able to control the kinetic temperature of the particles. As the particle temperature increased to a certain point the suspension melted and became liquid. By increasing the laser power still further, we heated the resulting liquid. Below we will discuss in detail the shear mode of in-plane particle motion in the solid and liquid states of our particle suspension.

To study the dispersion relation of the shear mode, we calculated the transverse current fluctuation spectrum $\mathcal{T}(k, f)$ [4,15,22], i.e., the transverse phonon spectrum. We first performed a discrete Fourier transform of $v_y(x, t)$ to yield $v_y(k, t)$, and then performed a fast-Fourier-transform algorithm to yield $v_y(k, f)$. To calculate $\mathcal{T}(k, f)$, we averaged $v_y(k, f)$ over time, as a proxy for computing an ensemble average. This was done using experimental sequences of 2048 frames (68.3 s duration) that were divided into subsequences of 128 frames (4.3 s duration). The subsequences overlapped, beginning at intervals of 36 frames (1.2 s). The delay time of 1.2 s was bigger than the neutral-gas damping time $\nu^{-1} = 1.1$ s. Finally, the square of $v_y(k, f)$ that was computed for each of these subsequences was averaged over all subsequences, yielding $\mathcal{T}(k, f)$. We did not apply subsequent smoothing or any other data-modification technique to $\mathcal{T}(k, f)$. As our particle suspension is two-dimensional, there are particles at many x values for any given range $\Delta x = a$; this allows us to calculate the values of wave numbers even at large values $k > \pi/a$. Because of unequal x spacing of the experimental $v_y(x, t)$ data, there is no k aliasing in the calculated fluctuation spectra.

The observed spectra of the shear waves are shown in Fig. 1. For the solid state, several Brillouin zones are clearly seen in Fig. 1(a). Here, the waves observed are a superposition of the waves in all possible directions with respect to the orientation of the lattice, due to its polycrystalline structure and a slow rotation in the experiment. In the liquid state, only a broadened long-wavelength part of the spectrum remained, Fig. 1(b). These spectra are similar to those observed in Ref. [12] using a different method of heating a 2D suspension of particles.

The cutoff of shear waves is revealed in the long-wavelength ($ka < 1$) portion of the spectra. To compute the dispersion relation from the spectra, we began with cross sections of $\mathcal{T}(k, f)$ as a function of k for a given f, as

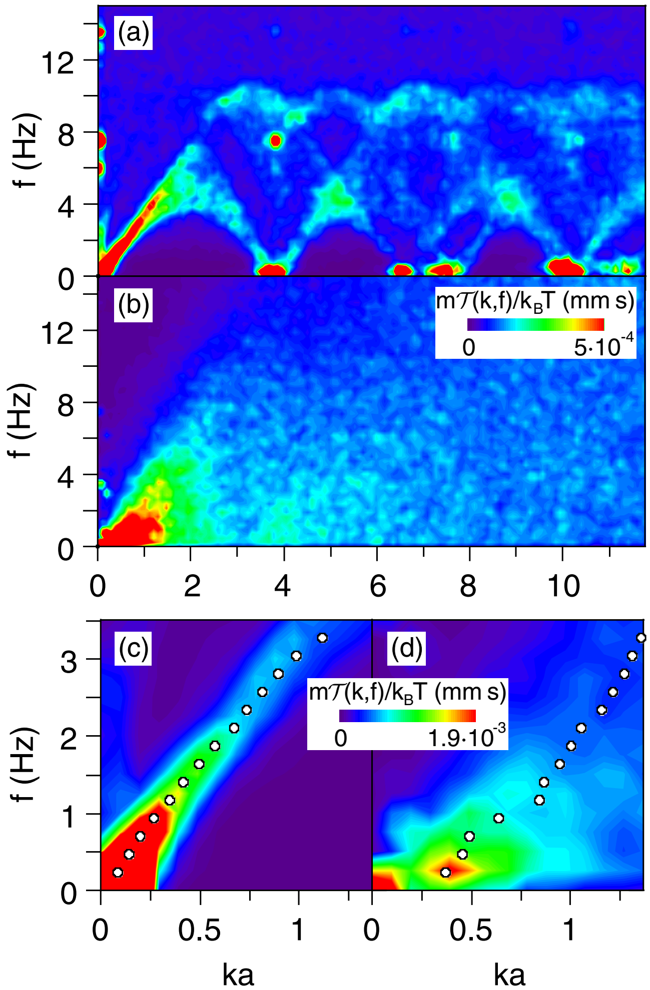

FIG. 1 (color online). Transverse current fluctuation spectra $\mathcal{T}(k, f)$ for two different states of a 2D dusty plasma: the solid state, where $\Gamma = 8300$ (a), (c) and for the liquid state, where $\Gamma = 73$ (b), (d). The panels (c) and (d) show the long-wavelength part of $\mathcal{T}(k, f)$; here, the color scale extends to 1.9×10^{-3}, so that the color scale in (d) does not saturate as in panel (b). Shear waves exist in a liquid dusty plasma only if their wave number is bigger than the cutoff wave number $k_c a = 0.27$, as shown in (d). The open circles in (c) and (d) indicate the shear wave's dispersion relation. The concentration of energy near $\omega = 0$ and $k_c a = 0$ in (d) indicates the presence of the sloshing mode, i.e., the motion of the particle suspension as a whole. The wave number resolution Δka is determined by the size Δx of the camera's field of view: $\Delta ka = 2\pi a/\Delta x = 0.091$.

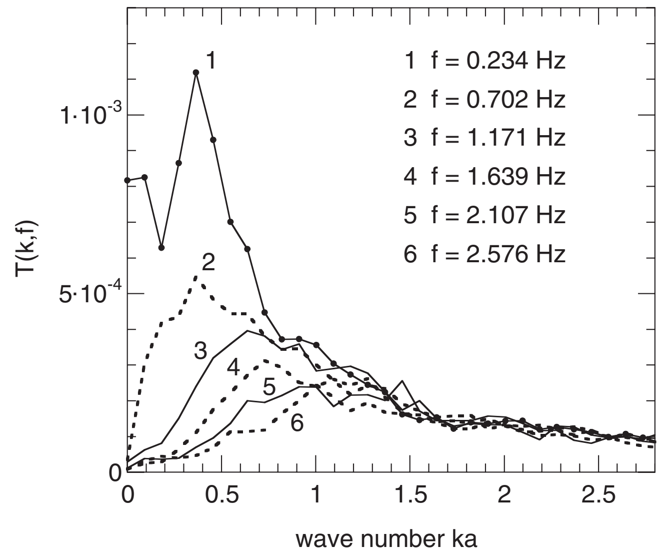

FIG. 2. Transverse current fluctuation spectrum $\mathcal{T}(k, f)$ from Fig. 1(d), evaluated at different frequencies f. The first moments of the peaks in curves like these were used to calculate the dispersion relations shown by open circles in Figs. 1(c) and 1(d).

in Fig. 2. We measured the k for the peak in each cross section by computing its first moment. Repeating for each frequency yields the dispersion relation $f(k)$, shown as open circles in Fig. 1. For the solid state, the dispersion relation in Fig. 1(c) passes through the origin at $f = 0$ and $k_c a \approx 0$, as expected. However, for the liquid state in Fig. 1(d), it does not. Instead, the dispersion relation reaches $f = 0$ at a finite cutoff wave number $k_c a = 0.27$.

The shear waves, therefore, can propagate in the liquid state of our particle suspension only if their wavelength is shorter than a critical value $2\pi/k_c$. This is well-known for simple liquids [14,15]. For strongly coupled plasmas, the cutoff wave number has been predicted in several simulations of liquids, both 3D [7,23] and 2D [4], but it apparently has never been observed experimentally.

The sloshing mode, i.e., motion of the particle suspension as a whole, appears in Fig. 1(d) as a concentration of energy at $f = 0$ and $k = 0$. This mode is unrelated to the shear wave, and it does not interfere with calculating the cutoff wave number k_c. Note that in this Letter the sloshing mode is less prominent than in our previous experiment of Ref. [17], where we externally excited compressional waves and observed the sloshing mode in the direction of excitation.

The experimental observation of the cutoff wave number k_c for the shear waves in a 2D liquid dusty plasma is a chief result of this Letter. Next, we discuss the dependence of k_c on the coupling parameter Γ, Fig. 3. We calculated k_c by beginning with the first four data points in the low-frequency portion of the dispersion relation [open circles in Fig. 1(d)] and fitting them to a straight line. Extrapolating this line to zero frequency yields our value of k_c. The error bars indicate the width of $\mathcal{T}(k, 0.234\ \mathrm{Hz})$ at the level of 75% of its peak value, as in Ref. [7]. The normalized cutoff values lie in the range of $k_c a = 0.16$–0.31 with perhaps a trend to increase for lower values of Γ. Our data generally agree with the molecular dynamics simulation of Ref. [4], where a cutoff wave number of $k_c a = 0.186$ was observed for the shear waves in a 2D Yukawa liquid, for $\kappa = 1$ and $\Gamma = 160$.

The damping mechanism that precludes the existence of the shear waves in a liquid below k_c is identified as

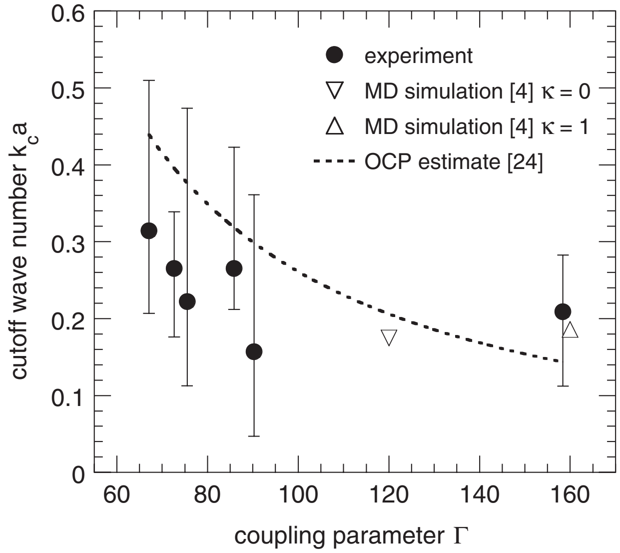

FIG. 3. Cutoff wave number $k_c a$ for shear waves as a function of coupling parameter Γ. We calculated k_c as the intercept with the k axis of the dispersion relation's low-frequency part, fitting $f(k)$ by a straight line for $f < 0.94$ Hz. The error bars indicate the width of $\mathcal{T}(k, 0.234 \text{ Hz})$ at the level of 75% of its peak value. Also shown is the cutoff wave number found in the simulation of Ref. [4] and an estimate $k_c a = 104\Gamma^{-1.3}$ for a 2D one-component-plasma approximation (OCP) ($\kappa = 0$) [24].

diffusional and other damping in Ref. [4] and viscous damping in Ref. [7]. Following Ref. [4], we calculated the diffusional-migrational time $\tau_{\mathrm{DM}} = 1/k_c C_T \approx 0.18$ s, where $C_T \approx 5.3$ mm/s is the shear wave's sound speed, for the conditions of Fig. 1(b) and 1(d). Note that τ_{DM} is significantly smaller than the neutral-gas damping time $\nu^{-1} = 1.1$ s; neutral-gas damping therefore did not obscure our observation of the cutoff wave number. In terms of the 2D nominal plasma frequency $\omega_{\mathrm{pd}} \equiv [Q^2/2\pi\epsilon_0 m a^3]^{1/2}$, $\tau_{\mathrm{DM}} \approx 13/\omega_{\mathrm{pd}}$, in agreement with Ref. [4].

Above k_c, shear waves exist, but they experience strong damping [7]. A combination of viscous and neutral-gas damping broadens the peak in the shear-wave spectrum, so that it has the width indicated by the error bars in Fig. 3. A similar increase in damping of shear waves in the liquid state was found in our experiments with sinusoidal wave excitation [13].

The partitioning of shear-wave energy into small wave numbers (though higher than the cutoff wave number k_c), as shown in Figs. 1(b) and 1(d), is expected for our particle suspension in the liquid state. Indeed, for any liquid shear waves disappear at large values of k, because the particles behave as if they were free, as in a perfect gas [15].

To summarize, we observed the shear waves in the solid and liquid states of a 2D Yukawa system (dusty plasma). For shear waves in a liquid, we verified the existence of the cutoff wave number. Its value was $k_c a = 0.16$–0.31 depending on the coupling strength. In other words, shear waves were only able to propagate in the liquid dusty plasma when their wavelength was smaller than (20–40)a. This result illustrates why it is difficult to observe shear waves in molecular liquids: their wavelength must be very short, perhaps shorter than a few tens of intermolecular spacings.

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