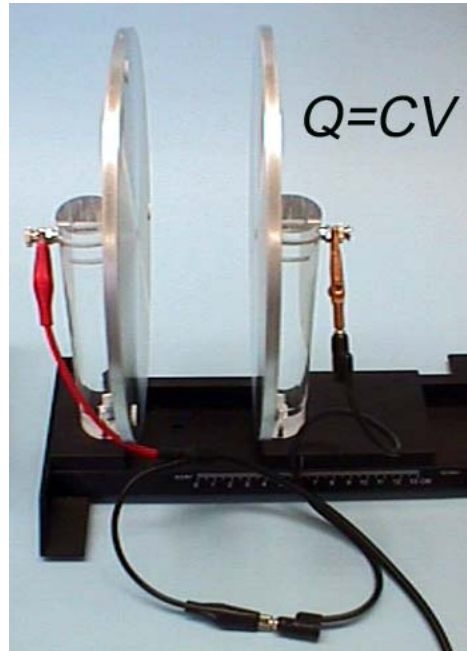# EXPERIMENT E4:

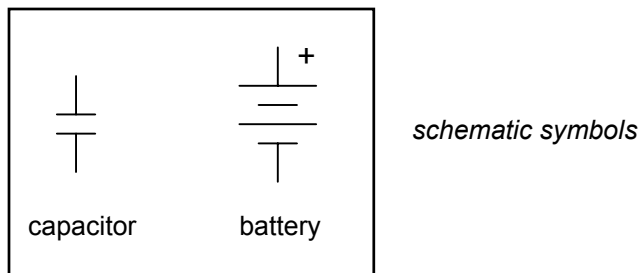# *Parallel-Plate Capacitor*

Objectives:

- *Scientific:*   Learn about parallel-plate capacitors
- *Scientific:*   Learn about multiple capacitors connected in parallel
- *Skill development:* Use curve fitting to find parameters from experimental data

Introductory Material

Capacitance is a constant of proportionality. It relates the potential difference V between two conductors to their charge, Q. The charge Q is equal and opposite on the two conductors. The relationship can be written:

$$Q = CV \qquad (4.1)$$

The capacitance C of any two conductors depends on their size, shape, and separation.

schematic symbols

One of the simplest configurations is a pair of flat conducting plates, which is called a "parallel-plate capacitor." Theoretically, the capacitance of parallel-plate capacitors is

$$C_P = \varepsilon_0 A / d \qquad (4.2)$$

where the subscript "P" denotes "parallel plate." Here, A is the area of one of the plates, d is the distance between them, and ε_0 is a constant called the "permittivity of free space," which has a value of 8.85×10^{-12} C^2 / N-m^2, in SI units.

Here is the basic idea of the experiment you will do. Suppose that you had a parallel-plate capacitor with the plates separated initially by a distance d_0, and you applied a charge Q_0 to the electrodes, so that they initially have a potential $V_0 = Q_0 / C_p$. Suppose that you then arranged for the two electrodes to be electrically insulated, so that the charge Q could not go anywhere. What would happen if you then increased the electrode separation d? The charge would remain constant, because it has nowhere to flow, whereas the capacitance would decrease, as shown in Eq. (4.2). Thus, the potential V will increase. If you measured V as a function of d, you would expect to find that the potential increases linearly with separation, like this:

$$V = V_0 d / d_0 \qquad (4.3)$$

If you plotted your experimental data and found that V is proportional to d, this would support the theoretical prediction of Eq. (4.2).

In an actual experiment, however, you will find that V and d are *not* proportional. Equation (4.3) must be improved, because it neglects the effect of additional capacitances in the setup. To measure the potential, we must have wires, or leads, connected to an electrometer, as shown in the sketch. The leads are conductors, and like any conductors they have a capacitance, which we will denote C_L. The circuit inside the electrometer also has a capacitance, which we will call C_E.

These capacitances are all in *parallel*, as shown in the sketch. When several capacitors are
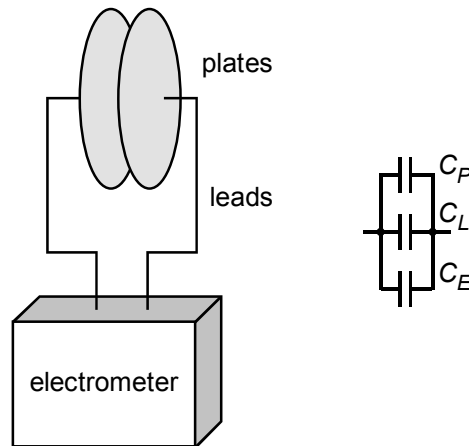
The capacitance C_P of the two plates
is in parallel with:
- the capacitance C_L of the leads and
- the capacitance C_P of the
  electrometer.

connected in parallel, they are equivalent to a single capacitor C_{tot}, which is the sum of all the components:

$$C_{tot} = \sum_i C_i \qquad (4.4)$$

You might think that the parallel plates, being the largest conductors in your setup, would have such a large capacitance that you can ignore the other capacitances. However, it turns out that this is not true. To see this, let's make a rough estimate. Consulting a reference book on electronics, one can find that the capacitance of a coaxial cable, such as you will use between your electrometer and the plates, is typically 1 pF for each cm of cable length. If the coax cable is 30 cm long, it will have a capacitance of approximately $C_L \approx 30$ pF. This ignores the additional capacitance of the two unshielded leads at the end of the cable. In the pre-lab questions, you will compute the capacitance of your parallel plates, when they are separated by a typical distance of 10 cm, and you will find a value that is comparable what we just estimated for the cable. Thus, we expect that the capacitance of the coax cable, by itself, is enough that we cannot use Eq. (4.3). We must include the effects of the other capacitances.

To replace Eq. (4.3) with an expression that includes the other capacitances, let us begin by combining Eq. (4.1) and Eq. (4.4), yielding:

$$Q = (C_P + C_L + C_E)\,V \qquad (4.5)$$

Expanding C_P using Eq. (4.2), we find:

$$Q = (\varepsilon_0 A / d + C_L + C_E)\,V \qquad (4.6)$$

Since we will measure the potential V, we would like to write an expression with V on the left-hand side:

$$V = \frac{Q}{\dfrac{\varepsilon_0 A}{d} + C_L + C_E} \tag{4.7a}$$

We can fit our experimental data for V as a function of d, to Eq. (4.7). As an alternative, we can re-write Eq. (4.7a) as

$$V^{-1} = \frac{(\varepsilon_0 A) d^{-1} + C_L + C_E}{Q} \tag{4.7b}$$

The form of Eq. (4.7b) has the advantage that it can be fit to the equation of a straight line.

$$V^{-1} = M d^{-1} + B \tag{4.8}$$

where the independent variable is d^{-1}, and the dependent variable is V^{-1}. Performing the fit will yield the slope M and intercept B. Using these coefficients, it can be shown, using a few steps of algebra, that

$$Q/A = \varepsilon_0/M \tag{4.9}$$

and

$$(C_L + C_E) = QB = A\varepsilon_0 B / M. \tag{4.10}$$

Note that our fit has two free parameters (M and B), whereas Eq. (4.7) depends on three parameters: Q, A, and ($C_L + C_E$). Of these three, the only one for which we have an independent measurement is the electrode area A. Therefore, we can use our fit parameters to yield measured values of the charge Q and the instrumental capacitance ($C_L + C_E$).

Finally, we discuss how the capacitance C_P of the parallel plates depends on the medium inserted between the plates. When there is not a vacuum between the two capacitor plates, the capacitance in Eq. (4.2) should be re-written as

$$C_P = K\varepsilon_0 A/d \qquad\qquad (4.11)$$

where *K* is the (dimensionless) dielectric constant of the material between the electrodes. Here is a table for some materials:

Material	Dielectric Constant *K*
Vacuum	1
Air	1.00054
Paper	3.5
Teflon	2.1
Glass	4.6
Water	80

As you can see, although we have been neglecting the dielectric constant of air, the error we make in doing so is quite small.

You might ask whether you could make more measurements in this experiment, using materials other than air. If you like, as a test you may try inserting a book or a stack of paper between the electrodes, after you have charged them. The potential across the electrodes should diminish. Books with glossy covers don't work well, because they are easily charged by handling them. A book with a rough cloth cover works well.

Pre-Laboratory Questions

(1) Calculate the area *A* of one electrode plate. Each plate is a circular disk of diameter 20 cm.

(2) Compute the capacitance C_P of the two plates in your experiment, when they are separated by 1 cm. Express your result in units of picoFarads (pF).

How to dress for this lab:

- Don't wear wool or synthetic materials such as nylon. These materials can disturb your experiment because they are easily charged by tribolectric effects.

- Cotton and cotton-blend clothes are ideal.

Equipment List

Parallel-plate capacitor, with adjustable inter-electrode spacing
6 V battery
100 M Ω charging probe*
1 Banana plug cable*, black, approx. 120 cm, with 1 alligator clip (for ground lead)
1 Banana plug cable*, black, approx. 60 cm, with 2 alligator clips (for battery)
Cable with BNC at one end, 2 alligator clips at other end (for electrometer)
Electrometer (Pasco ES-9054B)
Multimeter*
Caliper*
Computer*
Graph software (Graphical Analysis for Windows, or similar)

An asterisk indicates that an item is described in the Instrument Glossary.
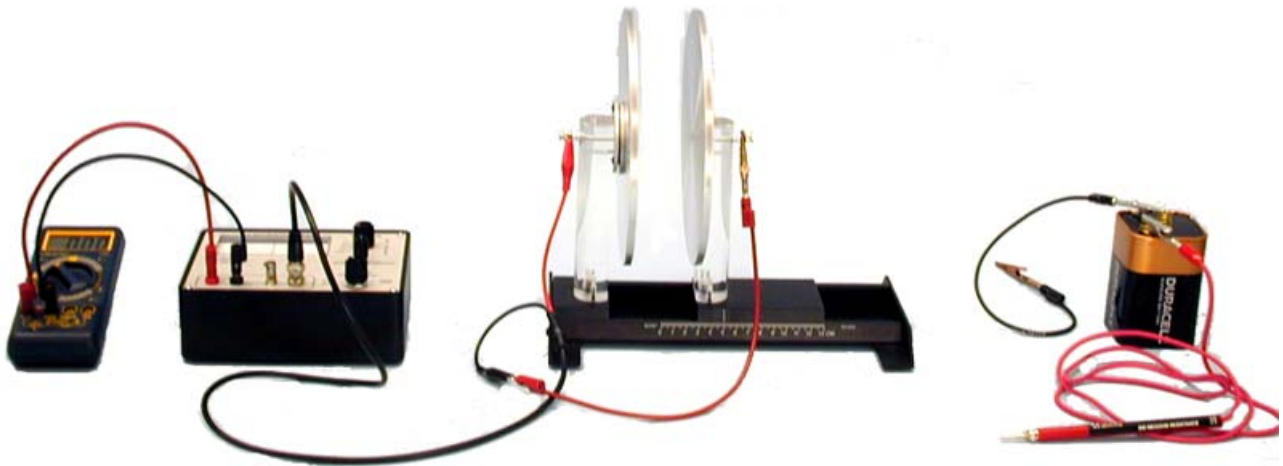
setup for parallel-plate capacitor experiment (caliper is not shown)

schematic diagram
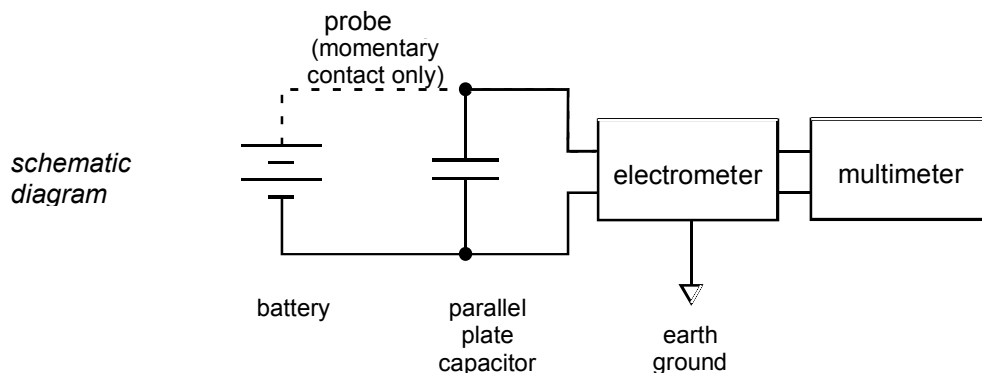
Experimental Procedure

Part I: Preparing the apparatus

If possible, avoid using a setup located in a high-traffic area of the room (for example, near a computer printer.) Your measurement may be disturbed by the electric charge on somebody walking by.

Examine your setup. Identify all the parts that are labeled in the setup diagrams, above.

6 V Battery. One terminal is connected to a 200 MΩ probe, the other to a banana plug cable. After charging the capacitor, put the cables down on the bench, as shown here, so that the charge on them does not disturb your measurement.

Be careful not to short-circuit the battery.

A short circuit is when a wire makes a closed path from one battery electrode to another. If you do create a short circuit, the battery will get hot and it will soon lose its charge and become worthless.

If you should discover that the battery is hot, immediately disconnect it.
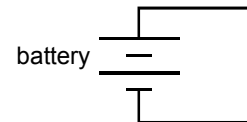
battery

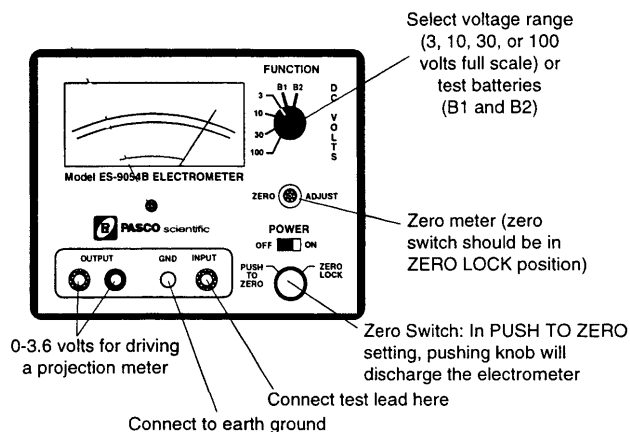DON'T DO THIS: this battery has been short-circuited.

Check two things on the mechanical setup of the capacitor plates:

- Verify that that the capacitor plates are parallel. Move the plates to the minimum separation. There are three plastic disks attached to one of the plates, so that the two metal plates cannot touch. If the two plates are adusted to be parallel, all three disks should make contact.[*]

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[*] If the plates are not parallel, ask your TA for help. The plates can be adjusted by turning the mounting screws, which press against a spring.

Select voltage range (3, 10, 30, or 100 volts full scale) or test batteries (B1 and B2)

Zero meter (zero switch should be in ZERO LOCK position)

Zero Switch: In PUSH TO ZERO setting, pushing knob will discharge the electrometer

0-3.6 volts for driving a projection meter

Connect test lead here

Connect to earth ground

To ground the two electrodes: press the "PUSH TO ZERO" button on the electrometer. This will momentarily short the two capacitor plates, removing any charge.

To charge the electrode, touch one of the capacitor plates momentarily with the 200 MΩ probe. This probe should be connected to one terminal of the battery. The other battery terminal should always be connected to the other capacitor plate.

- Verify that the scale for electrode separation is zeroed properly. Move the sliding electrode to any separation, for example 10 mm. Use the caliper to measure the separation. An accuracy of ± 0.1 mm or ± 0.2 mm is sufficient.[†]

Because you will measure distances with an accuracy of approximately ±0.2 mm, your best measurements of d will be at large distances, while your worst will be for small distances. Your apparatus allows you to adjust d as narrow as 1 mm, but the percentage error in your measurement would be too large at that separation. For that reason, we recommend making measurements for $d \geq 4$ mm.

To charge the capacitor, you will use a battery rather than a dc power supply. The battery is low voltage (6 V) and therefore poses no hazard for electrical shock.

When you connect the battery, in this experiment it doesn't matter which terminal you use, positive or negative, for the two leads. The polarity of the charge on the capacitor plates will depend on the way you connect the battery, but this will merely change the sign of your measurements of potential.

[†] If the zero is not adjusted, record the difference and correct your data later.

The electrometer is battery powered. Unlike instruments with a three-wire power cord plugged into an electrical outlet, battery-powered circuits do not usually have an earth ground. To provide a ground, your electrometer has a special terminal, which you should connect via a banana plug cable to an earth ground, such as the gas or air outlet by your lab bench.

For this experiment, you should adjust the electrometer so that it is on the 30 V scale. In this setting, when the meter needle points to its maximum value of "3", the instrument is reading 30 V. corresponds to 30V.

To ground the electrodes, so that they initially have zero charge, press the "PUSH TO ZERO" button on the electrometer. This is a momentary-contact switch that shorts the electrometer's two input leads. See the photo. While pressing this button, check whether the electrometer has been zeroed. If not, hold the button down while turning the "ZERO ADJUST" knob until the meter reads zero.

You will use the 200 MΩ probe to charge the electrodes. The reason we choose this probe is that it provides a suitable "*RC* charging time." With a capacitance *C* of typically 100 pF in this experiment, the product *RC* is 20 msec. This means that the final charge on the capacitor plate will be established during the final tens of milliseconds that the probe contacts the plate. This is a suitable time scale for the human hand to make contact with something.

The multimeter is used here to provide a digital display for the electrometer. It is not a required instrument for your measurements, but it will allow you to make your readings more accurately than is possible with the analog meter on the electrometer. Connect the output of the electrometer (two banana plugs) to the input of the multimeter (two more banana plugs), as shown in the photo. Choose the 20V DC scale on the multimeter, as shown in the photo.

A multimeter is used as a digital display for the electrometer. Set it on the 20 V DC scale. On some meters, DC is indicated by two lines, like this: ▭

The combination of using a 30V scale on the electrometer and a 20V scale on the multimeter will cause the multimeter to read one-tenth the actual value. Therefore, you will need to multiply your results by 10.
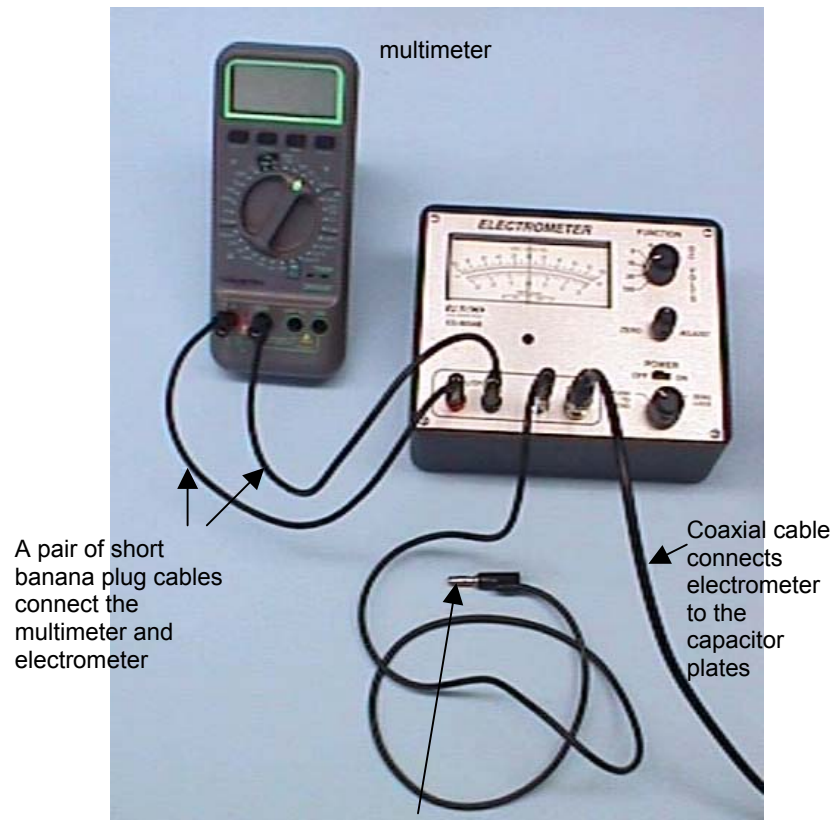
multimeter

A pair of short banana plug cables connect the multimeter and electrometer

Coaxial cable connects electrometer to the capacitor plates

Banana plug cable to connect electrometer to an Earth ground, such as a gas outlet. Sometimes the experiment can be performed successfully without this ground lead.

4-10

The cables and leads are important for this experiment. If you do not use the correct cables, or if you do not arrange them correctly on your table, you will find that the experiment does not work, due to the cable capacitance changing as you move the cables about. We have found good results using the cables and leads shown in the photographs below. If the cable you are provided is not identical to the one shown below, ask your TA for the correct cable:

Red alligator clip for fixed electrode

Red wire

Black wire

Coaxial cable

Alligator clip for movable electrode

Alligator clip (top) holding a banana plug (bottom)

- Before continuing, connect all the cables and leads. See the schematic diagram and the photograph of the setup near the beginning of this write-up.

Recommended placement of cables and leads.

Avoid moving the leads and cables after you begin recording measurements.

Part II: Measure potential V vs. separation d

In this portion of the experiment you will charge the capacitor, then measure the potential across the capacitor plates for various electrode separations.

1. First follow the procedure above, to check that the plates are parallel and the zero for the distance is properly calibrated.

2. Position the electrodes for a separation of 5 mm.

3. Ground the electrodes.

4. Because your measurement can be corrupted by charges on your body and clothes, you will need to stand at arm's length from the apparatus during the remainder of the experiment.

5. Apply a charge to an electrode using the two leads attached to the battery (one is a banana plug, the other is a 200 MΩ probe).

The charge that you have just now placed on the capacitor will remain there throughout the remaining measurement. This is a key physics concept for the experiment.

6. After applying the charge to the electrode, remove the leads from the electrodes, and perform steps 7 – 10 quickly, before the charge leaks off.

7. Measure the potential indicated by the electrometer, reading the value from the multimeter display.

8. Take the following precaution to avoid corrupting your measurement by extraneous charges:

    ▪ Place the leads on the bench, at least 30 cm away from the electrode plates.

    ▪ Put your hands on your lap.

    ▪ Make sure that nobody is walking by your setup.

9. Without disturbing the charge to the capacitor, move the sliding electrode to 6 mm.

10. Measure the potential.

11. Record an estimate for the ± uncertainty in your potential measurement. This should be based either the fluctuation level that you see on the meter, or how unsure you are of the reading if it was dropping rapidly due to water vapor in the room. (You should decide for yourself which of these two sources of uncertainty is more significant for your measurement, because it will vary from one day to the next, depending on the humidity.)

12. After recording the potential, move the sliding electrode back to the initial separation of 5 mm.

13. Repeat steps 5-12 for each of the following distances: 5, 6, 7, 8, 10, 12, 15, 20, 25, 30, 40, 50, 65, 80, 100 mm. You will apply a fresh charge each time, at a separation of 5 mm, and then change the separation. For the smaller separations, you should attempt to position the electrode with a tolerance of ± 0.2 mm. For separations larger than 8 mm, the tolerance is much less critical.

14. As a test, you and your partner should try moving your body and arms to and fro, to see if the electrostatic charge on your clothes affects the measured potential. If it does, you should repeat your measurements. To reduce your electrostatic charging, stand farther away, move less, and ground yourself by touching an earth ground, such as the ground terminal on the electrometer.

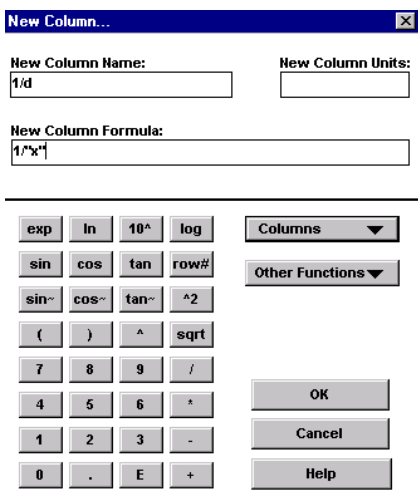## Part III: Plot your data for V vs. d and fit it

Plot the data by computer:

Use the software "Graphical Analysis for Windows" or a similar application.

- Enter your data in two columns, the first column for separation d, the second column for potential V.

- Generate a plot of V vs. d.

Note that the plot does not show a line passing through the origin, as suggested by Eq. (4.3). This is an indication of the effect of the instrumental capacitance, (C_L+C_E).

You will now prepare the data so that you can use Eq. (4.7b). You will need to carry out a procedure to compute the recipro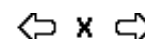cals of d and V. Here we explain how this is done using "Graphical Analysis for Windows." Click on the **f(x)** button, shown to the right, to create a new column. This will generate a window as shown at the left. Click in the box that says "New Column Formula," then type "1/" and then click select one of your existing columns of data from the "Columns" menu, for example "X". The formula should now read "1/"X"". Type a meaningful label like "1/d" in the box entitled "New Column Name" so that the plot axis will be labeled this way. Click "OK." A new column of data will appear in the "DATA TABLE WINDOW." To select this column for plotting, click on the item in the "GRAPH WINDOW" shown with two arrows, as shown to the right, and select the new column of data from the menu. Repeat this procedure with the vertical axis, to generate a column of data for $1/V$.

- Generate a plot of $1/V$ vs. $1/d$.

If the plot of appears to be a straight line, this verifies our model, which includes two key physical concepts:

- the scaling $C \propto 1/d$ for the capacitance of two parallel plates separated by a distance d

- the rule for adding capacitances in parallel, Eq. (4.4).

The intercept in this plot is non-zero, due to the instrumental capacitance, (C_L+C_E).

If you find that your results for voltage vs. distance are not as expected, one possible problem is that you may have used the wrong cable, or connected it incorrectly, or made a loose connection. If you moved the cables about during the measurement, or if you did not place the charging probe far from the apparatus when you put it down, that might also have corrupted your results. Another possible problem is electrostatic charge on your clothes. Re-read the instructions and re-examine the photographs carefully.

Fitting the data:

You now should fit the data to a straight line. It is recommended that you do this using a computer. If you use "Graphical Analysis," this is done as follows:

- click your mouse in the plot window.

- select "Automatic curve fit …" from the "Analyze" menu.

- Then choose a straight line, *M*x+B*.

Now that you have fit the data:

- Print the plot of 1/*V* vs. 1/*d* with the fit, displaying the values of *M* and *B*, and include this plot with your worksheet.

- Now select the columns for *V* and *d*, and plot *V* vs. *d*. Print this plot, and include it with your worksheet.

Part IV: Potential vs. separation without instrumental capacitance

In this part, you will use an extra instrument, with an op-amp amplifier that cancels the effect of the instrumental capacitance of the cable. The way it does this is by sensing the potential on the inner conductor of the cable and applying the same potential to the outer conductor, so that there's no charge accumulation between the two conductors.

You will make measurements similar to those in Part II, except that you will start with a plate separation of 100 mm, where you will apply a charge, and then you will reduce the separation before measuring the potential. (The reason for this is that otherwise the potential would be too high for your electrometer to measure.)

- Measure the potential at the following separations: 10 mm, 25 mm, 40 mm, 55 mm, 70 mm, 85, mm. Enter the values in the worksheet table, along with the value of the potential you applied at 100 mm.

- Plot *V* vs *d*.

  If the origin of the plot isn't (0,0), adjust it so that it is. In Graphical Analysis and many other kinds of plotting software, this is done by double clicking on an axis in the plot window, and then selecting an option to choose the minimum value for a plot's axis. (In Graphical Analysis, you can select "More X-axis options", then type "0.0" over the existing value for the "Left Limit.") You then repeat for the other axis. Be sure that the maximum value for the axis is chosen large enough to show all the data points.

- Fit the plot to a straight line passing through the origin: If you use "Graphical Analysis," this is done as follows:

  - ❏ Click your mouse in the plot window.

  - ❏ Select `Automatic curve fit` … from the `Analyze` menu.
  - ❏ Then choose a straight line, `M*x+B`.
  - ❏ Delete the `+B` from this expression, so that the remaining expression is `M*x`.

- In the Analysis Questions, you will discuss whether this result is consistent with Eq. (4.3).

- Be sure to turn off the battery-powered instruments

Part V: Clean Up

- Turn off instruments.

- Be sure to turn off the battery-powered instruments:
  - electrometer
  - multimeter.

- Unplug cables.

- Tidy the lab bench.

- Exit any running software.

Analysis Questions

Part III: Plot your data for V vs. d and fit it

1. Using Eq. (4.9), compute the charge that you applied to the electrodes. Be sure to include the units (C).

2. Using Eq. (4.10), compute the instrumental capacitance ($C_L + C_E$). Include the units.

3. Discuss briefly whether the instrumental capacitance is large. To do this, compute the ratio of the instrumental capacitance to C_P for a plate separation of 10 cm, and discuss in one or two sentences whether this ratio is large enough to significant affect your results.

Part IV: Potential vs. separation without instrumental capacitance

4. Discuss in one or two sentences whether your measurement is consistent with Eq. (4.3).