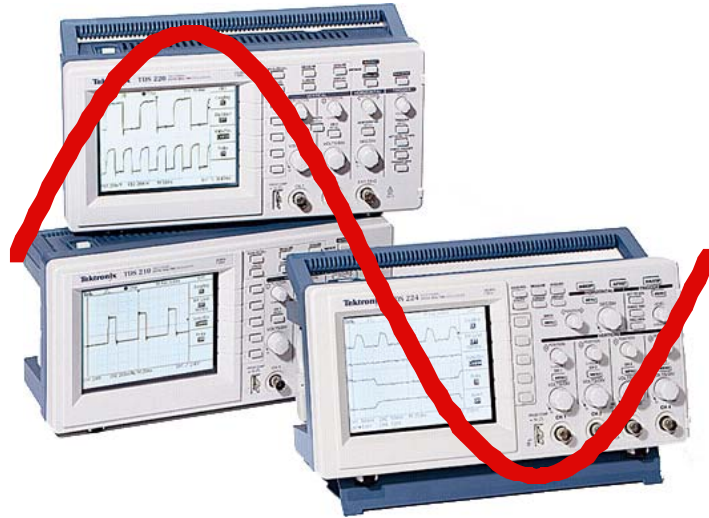


EXPERIMENT E11:

RLC Resonant Circuit



Objectives:

- Learn about resonance.
- Measure resonance curves for an RLC circuit.
- Investigate the relationships between voltage and current in circuits containing inductance and capacitance, when alternating voltages are applied.
- Test the frequency dependence of capacitive and inductive reactance.
- Measure the frequency response for an RC filter.
- Learn to wire a circuit.
- Learn to use an oscilloscope and function generator.

Introductory Material

A. The Capacitor and Inductor in AC Circuits

Inductance and capacitance are properties of circuit elements that depend on the geometry of the conductors and insulators of which they are composed. They are related to fundamental physical quantities, such as charge, potential difference and time according to the following relations:

$$V_c = \frac{Q}{C} \quad (11.1)$$

$$V_L = L \frac{\Delta I}{\Delta t} \quad (11.2)$$

To complete this list, Ohm's Law for a resistor is:

$$V_R = IR$$

In this experiment we will examine the characteristics of AC circuits which contain simple combinations of capacitors, inductors and resistors. Along the way we will define and measure some useful quantities which are similar to some things that are more familiar.

The units for capacitance C is farads, and for inductance L it is henries.

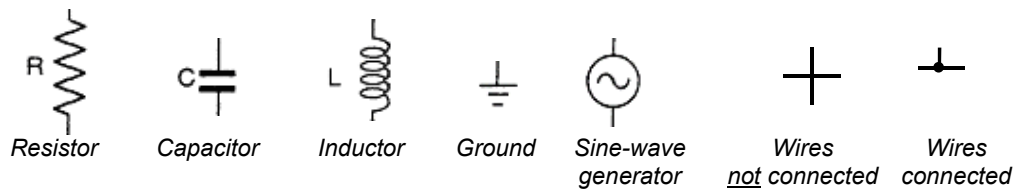


Figure 11-1. Symbols for schematic diagrams

An inductor is manufactured by winding a wire into a coil shape. The wire itself has some resistance, and therefore the inductor has not only an L , but also an R . In this experiment you will measure both the L and the R for your inductor.

B. Reactance and Impedance

Capacitors and inductors both can pass AC currents. The effect either has on an AC current flow is called **reactance** (X). These reactive components not only

limit the AC current flowing in a circuit, but also affect the time or phase relationships between the current and the various voltages present across the components. Therefore, in AC circuits the rules for combining these reactances must take into account both magnitudes and phase angles. The equivalent AC resistance (the quantity relating voltage and current) of a circuit containing reactances and resistances is called impedance (Z).

Let's compare the phase of the voltage and current in resistors, capacitors, and inductors:

- Resistor: The voltage across a resistor's terminals is at each instant directly proportional to the current through it. The voltage and current are said to be in phase with one another.
- Capacitor: The voltage and current are 90° (1/4 cycle) out of phase. The current through a capacitor depends on the time-rate-of-change of the voltage across the capacitor. *The voltage lags the current by 90° .* That is, if the current is a maximum at a certain instant, the voltage doesn't reach a maximum until a quarter period later.
- Inductor: The voltage and current are 90° out of phase, but in the opposite way, as compared to a capacitor. The voltage across an inductor's terminals depends upon the time-rate-of-change of the current through it. *The voltage leads the current by 90° .*

The magnitudes of the reactances at an applied frequency f are

$$\text{Capacitor} \quad X_C = \frac{1}{2\pi f C} = \frac{1}{\omega C} \quad (11.3a)$$

$$\text{Inductor} \quad X_L = 2\pi f L = \omega L \quad (11.3b)$$

The units of reactance is ohms. Note that the capacitive reactance becomes infinite at zero frequency (DC), but the inductive reactance approaches zero.

The expressions shown in Eq. (11.3) are the predicted values of the reactance. In the experiment, you will compute them from your measurements of f , C , and L .

C. Voltage-Phase Diagrams

It is convenient to display the instantaneous voltages across the elements in an AC circuit on a voltage-phase diagram. On such a diagram, the voltage across the resistor (V_R) is plotted on the $+x$ axis, the voltage across the inductor (V_L) is along the $+y$ axis, and the voltage across the capacitor (V_C) is on the $-y$ axis. See Figure

11-2. Notice that if the diagram is rotated counterclockwise about the origin, and we consider the projections of the vectors on the y axis, the "lead" and "lag" relationships discussed above are obeyed.

The quantity V is the applied voltage and at all times must be the vector sum of V_L , V_R , and V_C . For our diagram in Figure 11-2, we have chosen the particular time for which the voltage across the resistor (V_R) is taken to have zero phase angle. This is convenient because this voltage is in phase with the current throughout the circuit. Thus we have a "snapshot" of all the voltages at a particular instant. As time progresses, these rigid vectors can be thought of as rotating, and it is their projections on the y axis which are the sinusoidally varying quantities which we see on an oscilloscope. (You may want to read through this last paragraph again.)

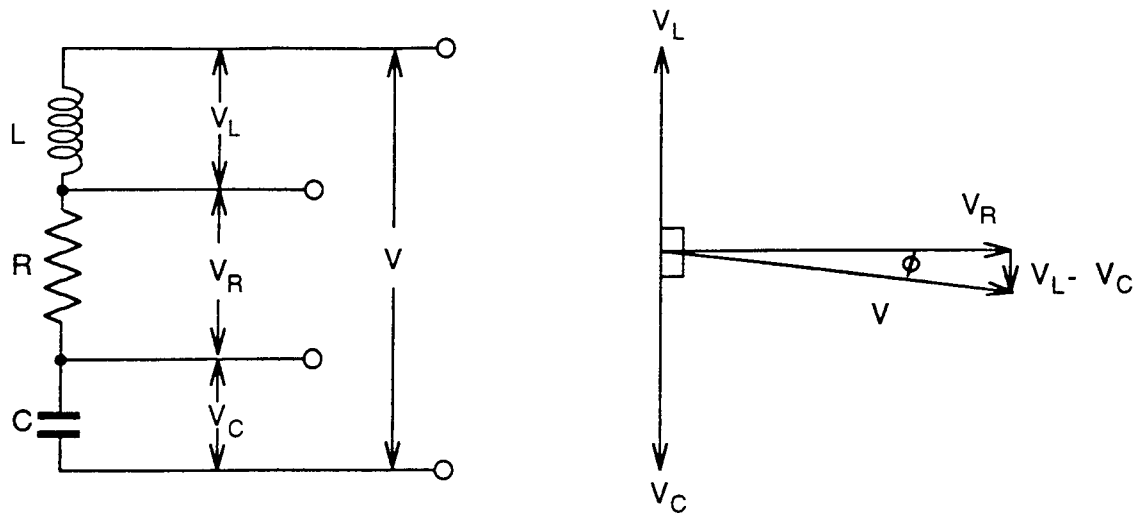


Figure 11-2. Series RLC circuit and voltage relationships.

All of the relationships we will be investigating can now be derived from the appropriate voltage-phase diagram. For example, it is a simple task to apply the Pythagorean theorem and find the resultant (or applied) voltage V ,

$$V = [V_R^2 + (V_L - V_C)^2]^{1/2} \quad (11.4)$$

The individual voltages can be written in terms of the current and the appropriate AC impedance,

$$V_R = IR \quad (11.5)$$

$$V_C = IX_C \quad (11.6)$$

$$V_L = IX_L \quad (11.7)$$

$$V = IZ \quad (11.8)$$

Inserting these quantities into Eq. (11.4) we obtain

$$IZ = [I^2 R^2 + (IX_L - IX_C)^2]^{1/2} \quad (11.9)$$

Dividing both sides by the current I , the total impedance is

$$Z = [R^2 + (X_L - X_C)^2]^{1/2} \quad (11.10)$$

D. Phase Angles

Figure 11-2 can also be used to obtain the phase angle between the voltage V (which might be the applied function generator voltage) and any of the other voltages. Between V and V_R the phase ϕ is seen to be

$$\tan \phi = \frac{V_L - V_C}{V_R} = \frac{IX_L - IX_C}{IR} = \frac{X_L - X_C}{R} \quad (11.11)$$

Figure 11-3 shows the circuit and voltage-phase diagram for the series RC circuit. For this circuit we have

$$V = [V_R^2 + (-V_C)^2]^{1/2} = I [R^2 + (1/\omega C)^2]^{1/2} \quad (11.12)$$

$$\tan \phi_C = \frac{V_R}{V_C} = \frac{IR}{IX_C} = \frac{R}{\frac{1}{\omega C}} = R\omega C \quad (11.13)$$

$$\tan \phi_R = \frac{V_C}{V_R} = \frac{IX_C}{IR} = \frac{\frac{1}{\omega C}}{R} = \frac{1}{RC\omega} \quad (11.14)$$

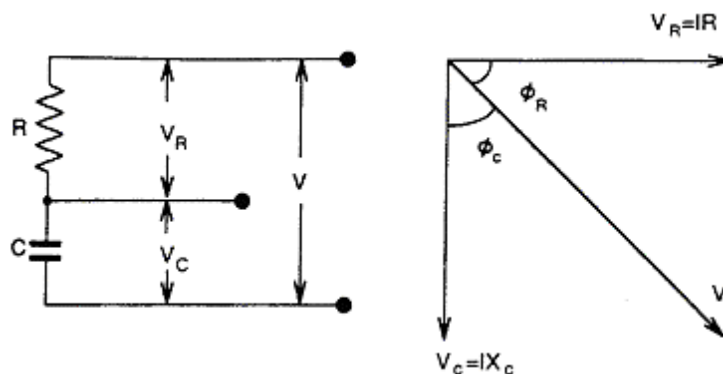


Figure 11-3.

Left: Series RC circuit

Right: Voltage relationships in this circuit.

E. Frequency response of resonant RLC circuit

Here will find the voltage across the resistor, V_R , as a function of frequency, for the RLC circuit shown in Figure 11-2. In the experiment will plot these quantities, and you will find a resonance, as shown in Figure 11-4.

Using Eq. (11.5), we see that $V_R = IR$. Therefore, we need an expression for the amplitude of the current I passing through the circuit. Using $V = IZ$ from Eq. (11.8), and substituting Eq. (11.10), we find

$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} \quad (11.15)$$

Substituting for X_L and X_C from Eq. (11.3), we find

$$I = \frac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} \quad (11.16)$$

The numerator in Eq. (11.16) is the amplitude of the sinusoidal voltage that is applied by to the circuit. Note that this expression has a resonance, i.e., a maximum value, at the frequency where $(\omega L - 1 / \omega C) = 0$. In other words, there is a resonance at frequency

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad (11.17)$$

or

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \quad (11.18)$$

In general, a “resonance” means that an amplitude becomes very large. In this case, it is the current through the circuit that becomes very large at the resonance frequency. In fact, if there were no resistance R , the current would become infinite. Using Eq. (11.5), we can rewrite Eq (11.18) to arrive at our final result

$$V_R / V = \frac{1}{\sqrt{1 + (\omega L - \frac{1}{\omega C})^2 / R^2}} \quad (11.19)$$

This expression is plotted in Figure 11.4.

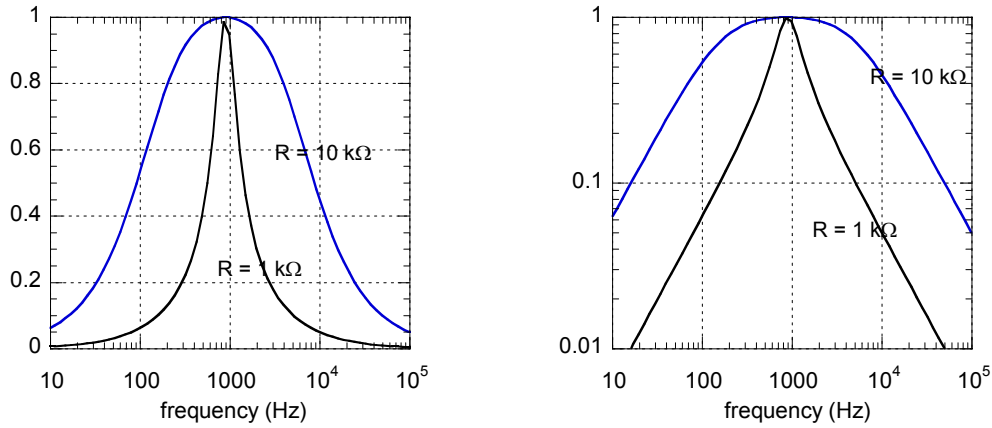


Figure 11-4. Frequency response of RLC resonance circuit, from Eq. 11.19.

The voltage V_R measured across the resistor of the RLC series circuit are predicted to look like this. Two curves are shown: one for $R = 1 \text{ k}\Omega$ and $10 \text{ k}\Omega$. Otherwise $L = 0.32 \text{ H}$ and $C = 0.1 \text{ }\mu\text{F}$ are the same for the two curves. Note that a lower resistance (less dissipation) gives a narrower resonance. If a circuit has a narrow resonance, it is said to have a "high Q ." In this case, the circuit with $R = 1 \text{ k}\Omega$ has a higher Q than the circuit with $R = 10 \text{ k}\Omega$. (The two graphs here show the same data; they differ only in the way the vertical axis is plotted, linear vs. logarithmic.)

Equipment List

Digital Storage Oscilloscope	Two resistors (nominal values 1 k Ω and 10 k Ω)
capacitor (nominal value 0.1 microfarad)	Inductor (nominal value 0.32 henry)
Multimeter, with Hz & capacitance measurement capability	Function generator (Wavetek)
BNC to banana adaptor	

Note: TA's should require ID cards to check out the oscilloscopes and the multimeters.

Pre-Laboratory Questions

- (1) Suppose that you connect an inductor ($L = 0.32$ H) and capacitor ($C = 0.1$ μ F) in series. Calculate their resonance frequency f_o , in Hz. (Show your work.)
- (2) Suppose that the same inductor and capacitor as in (1) are now connected in series with a resistor $R = 10$ k Ω . Calculate the phase angle for an applied frequency $f = 60$ Hz. (Show your work.)
- (3) When you adjust the "treble" knob on a stereo, are you adjusting a "high-pass filter" or a "low-pass filter"? Explain in one or two sentences why.
- (4) Is a resonance sharper when there is more dissipation or less?

Experimental Procedure

Part A: Circuit elements

Actual values of resistance for resistors, and capacitance for capacitors differ from the value that the manufacturer stamps on them because of “manufacturing tolerances.”

- Use the multimeter to measure and record the DC *resistance* R of the resistor. The two test leads (wires) should be plugged into the multimeter terminals labeled Ω and COM.
- Use the multimeter to measure the *capacitance* of the capacitor. Depending on the multimeter that you use, you will either:
 - insert one test lead into the terminals labeled COM and the other test lead into the terminal labeled either F or Cx
 - Or use special holes in the multimeter labeled either F or Cx. See Figure 11-5 (b).
- Record the value of C .
- Compare to the value printed on the capacitor by calculating a percentage difference. (Most electronic components like capacitors have a tolerance, or precision, of 20% or better.)

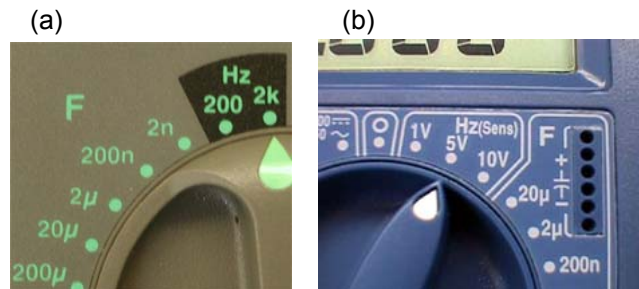


Figure 11-5. Some digital multimeters can measure frequency and capacitance. The appearance will vary from one model to another. Ask your TA if you are unsure how to connect your multimeter to the capacitor.

Inductors are not ideal; they can have a finite DC resistance.

- Use the multimeter to measure and record the DC resistance of the inductor, and record the value.



Figure 11-6.
Inductor

Exp. E11: RLC Resonant Circuit

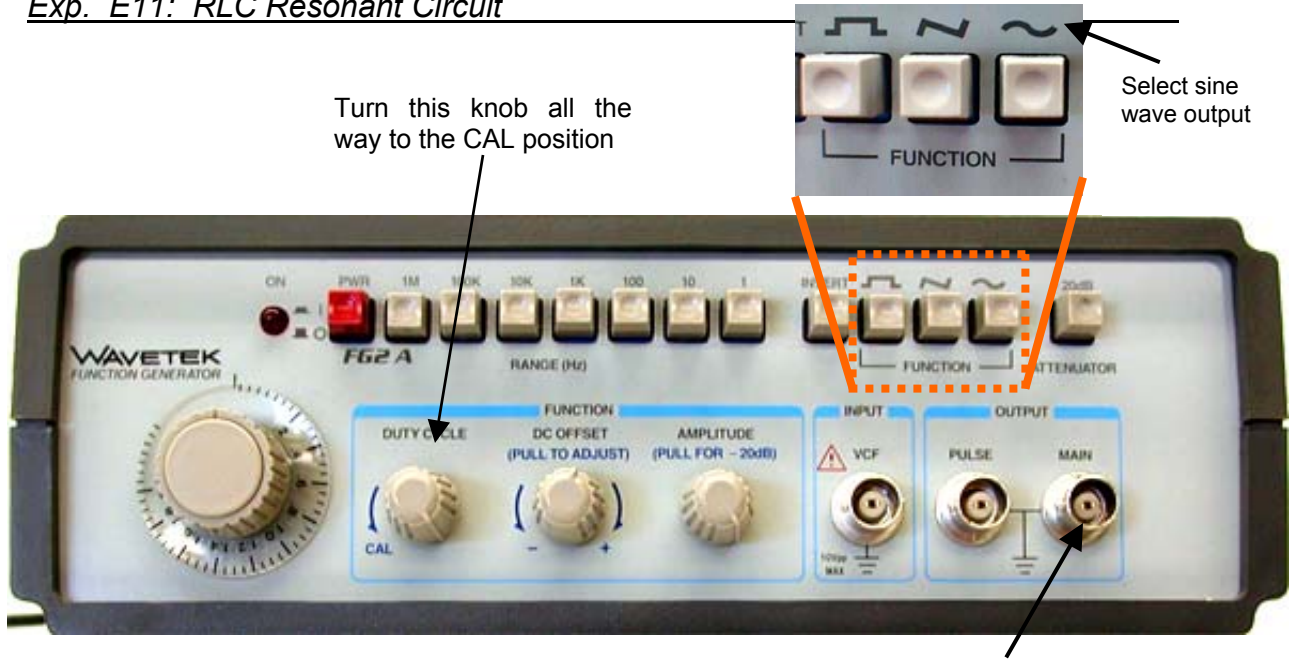


Figure 11-7. Function generator

This BNC jack provides the output

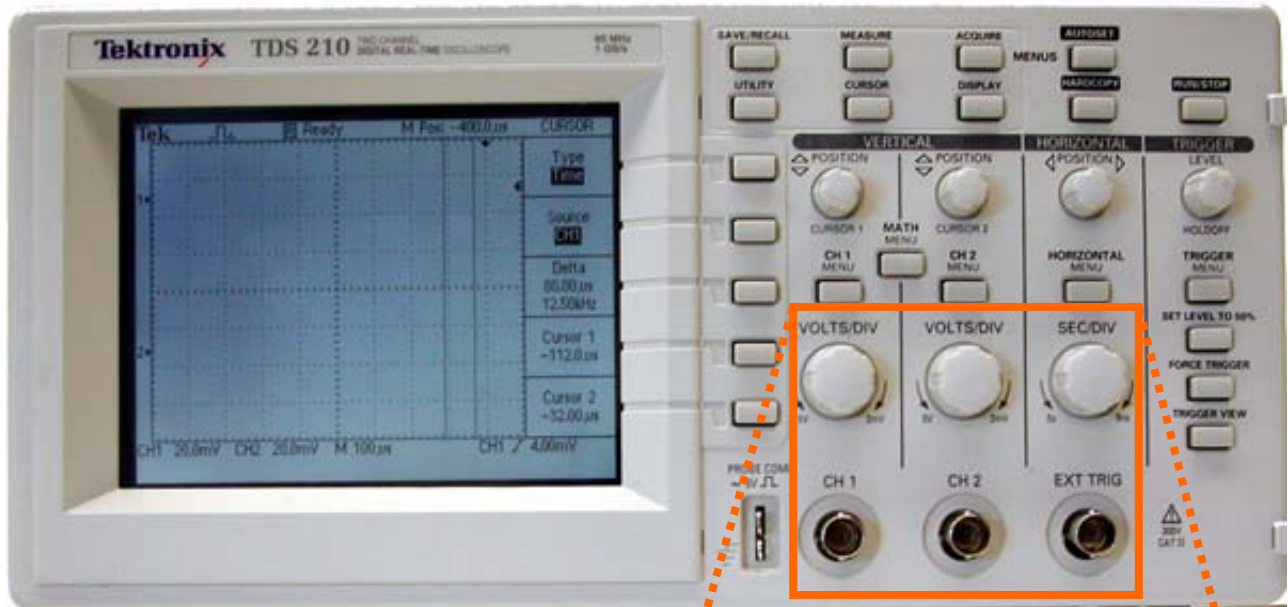


Figure 11-8. Digital Storage Oscilloscope.

The display will show the voltage on the vertical axis and time on the horizontal axis. Your two inputs signals will be connected to the two inputs, CH1 and CH2. You can adjust the vertical scale by turning the VOLTS/DIV knob, and you can adjust the horizontal scale by turning the SEC/DIV knob. The CURSOR button



provides precise measurements.



Part B: AC Voltages

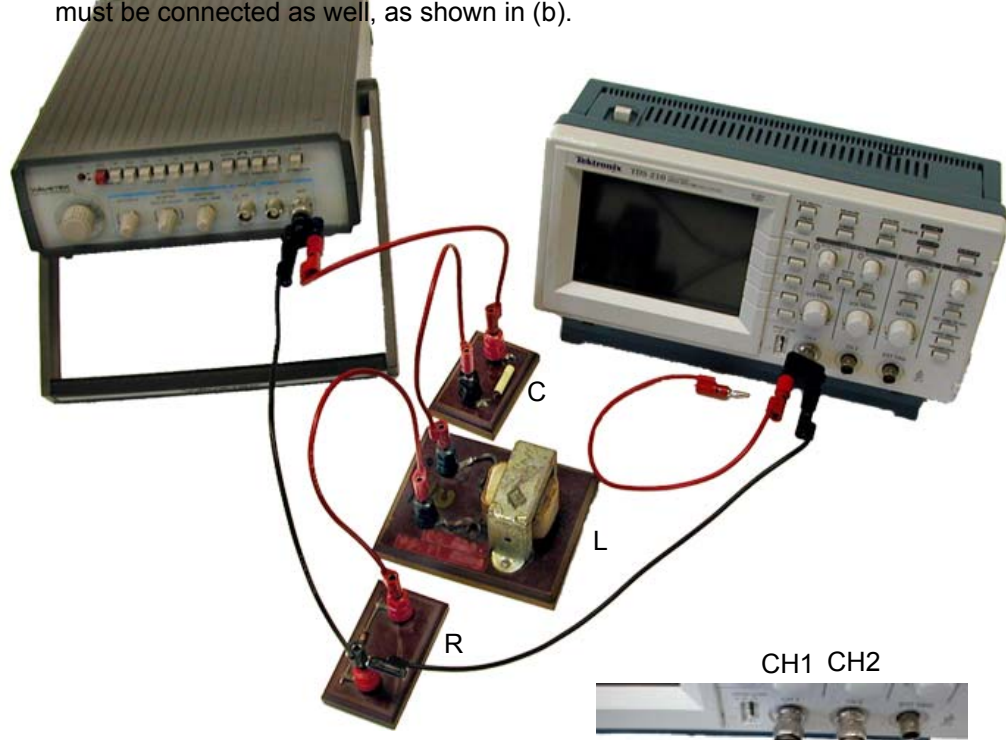
- (1) Examine the function generator, as shown in Figure 11-7 and the digital storage oscilloscope in Figure 11-8.
- (2) Connect the function generator, oscilloscope, and components as shown in Figure 11-9. Note that part (a) of this figure shows an incomplete setup, whereas (b) is a complete setup. You must connect the oscilloscope inputs as shown in (b). Use an $R = 10\text{ k}\Omega$ resistor.

To see how the oscilloscope and function generator work, perform the following test:

- Select a *sine* wave output for the function generator. Adjust its frequency to approximately 1 kHz by pushing the 1 kHz range button and turning the dial to 1.0.
- Press the “CH1 menu” button. Check that the following settings are selected:
 - Coupling = AC
 - Volts/Div = Coarse
 - Probe = 1X
- Press the “MEASURE” button.
- Press the “AUTOSET” button to cause a waveform to display.
- Note that two waveforms are displayed. One of them is CH1, the other is CH2. To distinguish them, look at the label on the left side of the display. If one of the waveforms is missing, press the CH1 or CH2 button to make it appear.
- If one of the waveforms looks like a square or triangle wave, check the settings of the function generator and set it for a sine wave. If it doesn't look like a good wave at all, check that you have hooked up the wires correctly.
- Push the “MEASURE” button on the scope.
- Note that on the right side of the screen there is a numerical display of the peak-to-peak amplitude and frequency of the waveform. This feature works correctly only if >1 full cycle of the sine wave is displayed.
- Vary the amplitude and frequency of the function generator, and observe the change of the scope display. After changing the frequency significantly, push the scope's “AUTOSET” button to see how this readjusts the display so that >1 full cycle is shown.
- Hereafter, when you measure frequency, use the value displayed for CH1, because sometimes the signal for CH2 will be too weak to measure frequency.

Exp. E11: RLC Resonant Circuit

(a) This setup is incomplete, you must connect CH1 and CH2 of the oscilloscope as shown in (b). The loose banana plug shown in this photo must be connected to the function generator output, as shown in (b), in order to measure V_R ; and CH2 must be connected as well, as shown in (b).



(b)

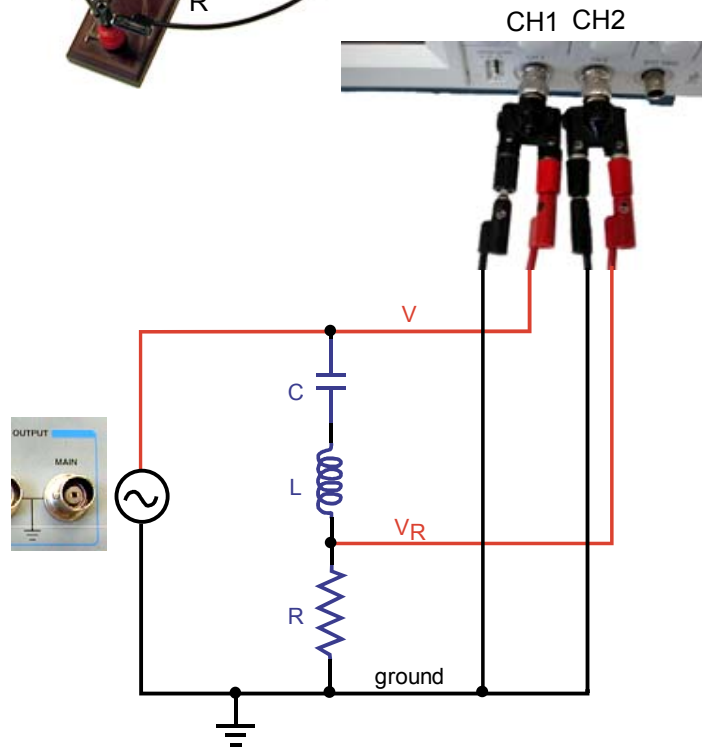


Fig. 11-9. Configuration for procedure B2.

(a): Photo of partial setup. Here the CH1 input of the scope is shown when it is not yet connected, to help keep the photo uncluttered.

(b): Schematic diagram of entire setup. Note that the entire bottom line of the circuit is connected to ground. Use scope input CH1 to measure the voltage V applied by the function generator and CH2 to measure the voltage V_R across the resistor.

3. Adjust the function generator to deliver approximately 10.0 volts peak-to-peak (pp) at 100 Hz. This is the applied voltage V .
4. Measure V_R as follows:
 - **Do not change** the function generator settings.
 - Push the “AUTOSET” button each time after changing the function generator’s frequency significantly. If you don’t see >1 full cycle of the sine wave, push the “AUTOSET” button again.
 - Record the peak-to-peak voltage for V_R .
 - Repeat for all the frequencies listed in the worksheet.
5. Repeat the measurements for step 4, this time using $R = 1\text{ k}\Omega$
6. For $R = 1\text{ k}\Omega$ only:
 - Vary the frequency until you find the maximum value of V_R . This is the resonance frequency f_0 . Record this value and the corresponding V_R in the worksheet.
 - Record values of V_R for $f = f_0 - 300\text{ Hz}$, $f_0 - 100\text{ Hz}$, $f = f_0 + 100\text{ Hz}$, and $f = f_0 + 300\text{ Hz}$.
7. Using computer software such as “Graphical Analysis”, plot V as a function of frequency. Include all the data from steps 4 –6, using two separate columns for $R = 1\text{ k}\Omega$ and $R = 10\text{ k}\Omega$. Make a plot, as shown on the left side of Figure 11-4, with a linear vertical axis and a logarithmic frequency axis.

Label the axes appropriately, including units, and print the plots.

Part C: AC Reactance

In this Part you will carry out some calculations of reactance, using your data from the previous Part for V_R and some new data you will take for V_L and V_C .

- Use the $R = 10\text{ k}\Omega$ resistor for this part.
1. Measure V_L :
 - Interchange the inductor and resistor (see the upper part of Figure 11-10.)
 - Measure V_L at the two frequencies listed in the worksheet.

2. Measure V_C :

- Interchange the inductor and capacitor (see the lower part of Figure 11-10)
- Measure V_C at the two frequencies listed in the worksheet.

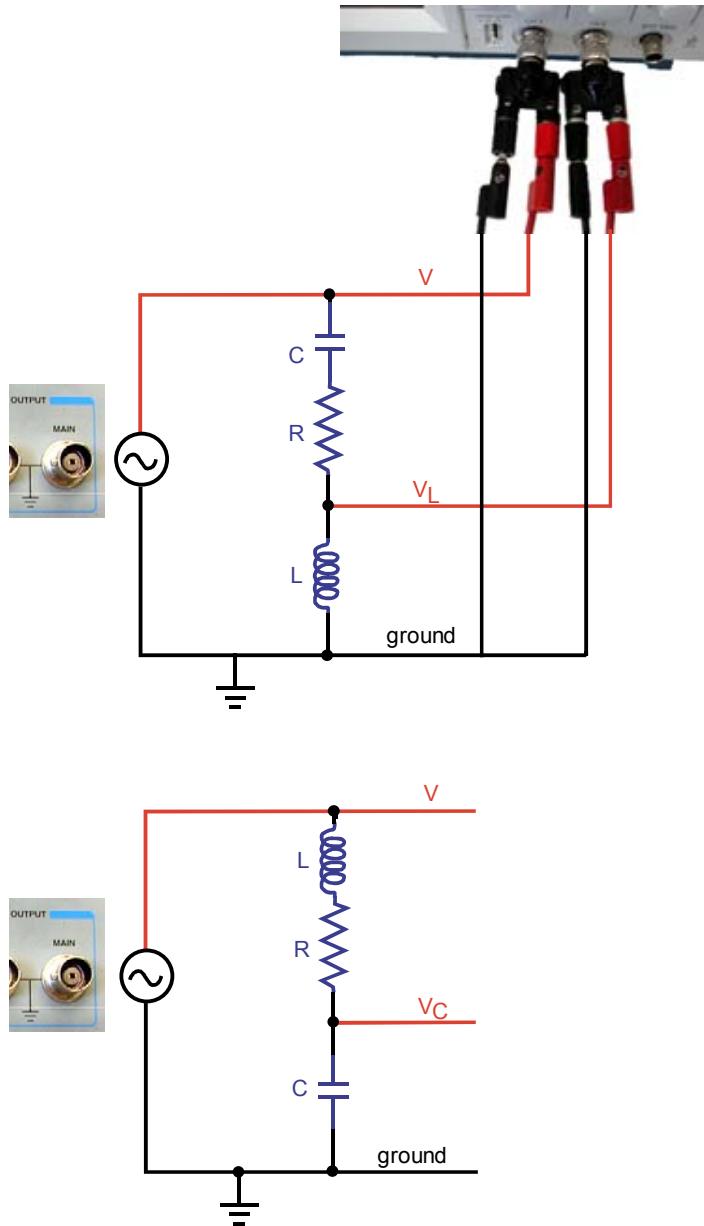


Figure 11-10. Schematic diagrams:

Top: For procedure C1 to measure V_L

Bottom: For procedure C2 to measure V_C (scope connection not shown)

The reactances X_C and X_L at a frequency f can be determined from the voltages that you measured. A short development is necessary to see how this may be done.

The same current I flows through all three components. From Eq. (11.5), (11.6), and (11.7) we have:

$$I = \frac{V_R}{R} = \frac{V_C}{X_C} = \frac{V_L}{X_L} \quad (11.22)$$

So that

$$X_L = \left(\frac{R}{V_R} \right) V_L \quad (11.23)$$

$$X_C = \left(\frac{R}{V_R} \right) V_C \quad (11.24)$$

4. Use Eq. (11.23) and (11.24) to calculate the measured values of the reactance from your measurements of V_R , V_L , and V_C .
5. The *predicted* reactances are calculated from Eq. (11.3) using:
 - the value of L marked on the component
 - the value of C that you measured with the multimeter,
 - the frequency f that you measured.
6. Fill in the table in the worksheet.

Part D: RC filter and Phase Angle Measurement

RC filters are widely used in electronic circuits. The resistor and capacitor are connected in series, and the output is taken across either the resistor or the capacitor, depending on whether you want low-pass or high-pass filtering:

- A “*high-pass filter*” permits only *high* frequencies to pass through to its output, and it attenuates *low* frequencies.
- A “*low-pass filter*” permits only *low* frequencies to pass through to its output, and it attenuates *high* frequencies.

You are familiar with filters like these if you have ever adjusted the “bass” and “treble” on a stereo.

Exp. E11: RLC Resonant Circuit

In this Part, you will find the frequency response, i.e., the output amplitude vs. frequency, for a filter. The customary way of doing this is to plot the output amplitude normalized by the input amplitude on the vertical axis, and the frequency on the horizontal axis.

We will also use this circuit to learn how to measure the phase angle between two sine waves of the same frequency. The method is called the sine wave displacement method. The reference sine wave (function generator output) is used to trigger the oscilloscope so that exactly one cycle of the wave fills the screen. The signal sine wave is then viewed and its displacement Δt along the horizontal axis (as illustrated in Figure 11-11) gives the phase shift according to

$$\phi = \frac{\Delta t}{\tau} 360^\circ \tag{11.25}$$

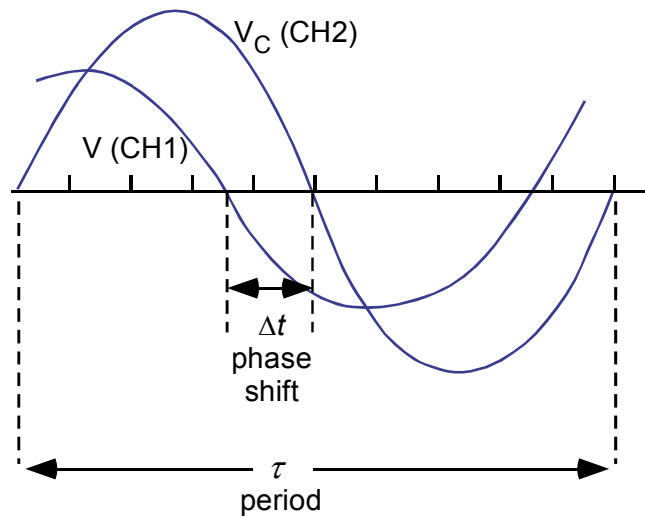


Figure 11-11. Determination of the phase shift ϕ for Eq. (11.25).

1. Connect the oscilloscope as follows:
 - (a) Connect the CH1 input to measure the voltage applied by the function generator, by connecting its center connector to the point labeled "V" in Figure 11-12. Use the $R = 10\text{ k}\Omega$ resistor for this part.
 - (b) Connect the CH2 input to measure the voltage appearing across the capacitor, by connecting its center pin to the point labeled " V_C " in Figure 11-12. Adjust the vertical position so that both traces are centered about the horizontal midline.

Adjust the SEC/DIV control to give at least 1 1/2 full periods (2 complete humps) of displayed wave.

 - (c) *To find the phase*, determine the time Δt as shown in Figure 11-11, and calculate the corresponding angle ϕ .
2. Fill in the table in the worksheet for the RC circuit amplitude and phase. (You will record values for phase for only three frequencies, as listed in the worksheet.)
 3. Prepare a plot of the amplitude response vs frequency, i.e., V_C / V vs. f , using log-log axes. Frequency should be the horizontal axis.

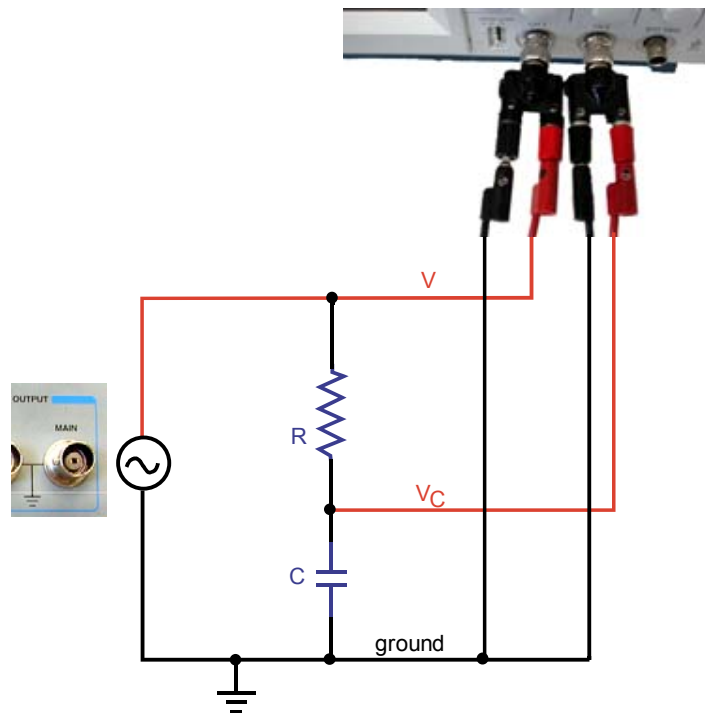


Figure 11-12. RC filter circuit for Part D.

Part E: Clean Up

- Turn off all instruments.
- Verify that the battery-powered multimeter is turned off.
- Unplug cables.
- Tidy the lab bench.
- Exit any running software.

Analysis Questions

Part B: AC Voltages

- (1) Calculate the expected value of f_0 using the values of C that you measured with the multimeter and the value of L that is marked on the component. Compare to the value of f_0 that you measured (for $R = 1 \text{ k}\Omega$) by calculating the percentage difference.

Part C: AC Reactance

- (2) As the frequency is increased, what trend (increasing or decreasing) do you expect for X_L and X_C ? Are your experimental results, for the two frequencies you measured, consistent with this?

Part D: RC filter and Phase Angle Measurement

- (3) Is the circuit you tested a “low-pass” or a “high-pass” filter?