Simulation of Three-Dimensional Dusty Plasmas

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Abstract—**The structure and dynamics of dust particles in a 3-D dusty plasma is characterized using a Langevin molecular dynamics simulation with a Yukawa potential. Conditions are set appropriate for a liquid-like strongly coupled plasma. The positions of dust particles are shown in an image. The thermal motion of particles is decomposed into the longitudinal wave spectrum, showing a distinctive dispersion relation.**

Index Terms—**Dusty plasmas, liquids, numerical simulation, waves.**

WE USE the Langevin dynamics simulation method of [1] to perform new simulations to characterize structure and waves in a 3-D dusty plasma. The equation of motion

$$m_p \ddot{\mathbf{r}}_j = -\nu_g m_p \dot{\mathbf{r}}_j + \zeta_{gj}(t) - \nabla \sum_l \phi_{jl} - \nabla \Phi$$

for the jth particle includes electrostatic forces due to particle–particle interaction ϕ_{jl}, a confinement potential Φ, gas friction $-\nu_g m_p \dot{\mathbf{r}}_j$, and random Brownian force ζ_{gj} due to gas molecules. Here, m_p is the particle mass and ν_g is the gas friction constant. We model the interparticle potential as a pairwise Yukawa potential, $\phi_{jl}(r_{jl}) = (Q^2/4\pi\varepsilon_0 r_{jl})e^{-r_{jl}/\lambda_D}$ for identical dust particles of charge Q with a screening length λ_D due to the electrons and ions. The collection of dust particles can be characterized by two dimensionless parameters: the Coulomb coupling parameter $\Gamma = Q^2/4\pi\varepsilon_0 a k_B T_d$ and the screening parameter $\kappa = a/\lambda_D$. Here $a = (3/4\pi n_d)^{1/3}$ is the Wigner-Seitz radius, n_d is the number density of dust particles, and T_d is the particle kinetic temperature. For the coupling parameter $\Gamma > 1$, the dust component is said to be strongly coupled, and dust particles can self-organize like atoms in a solid or liquid and sustain waves [2], [3].

Our simulation parameters are for the PK-4 instrument [4]. We use microsphere dust particles of radius 3.43 μm and $m_p = 2.55 \times 10^{-13}$ kg, with neon gas at 50 Pa pressure and 0.03 eV temperature so that $\nu_g = 51$ s^{-1}. We assume $Q = -8520e$, $n_d = 3 \times 10^4$ cm^{-3}, and $\lambda_D = 8.3 \times 10^{-3}$ cm. The characteristic interparticle distance is $a = 0.020$ cm, so that $\kappa = 2.4$. The characteristic time for particle motion is $\omega_p = 157$ rad/s, where $\omega_p = (Q^2 n_d/\varepsilon_0 m_p)^{1/2}$.

We chose $T_d = 8.3$ eV, corresponding to $\Gamma \approx 63$. For these values of Γ and κ, the collection of dust particles is predicted to behave like a liquid, according to the phase diagram of the Yukawa system [5]. We simulate $N = 12\,800$ particles in a 3-D rectangular volume defined by a flat-bottomed confining potential Φ.

Results shown in Fig. 1(a) reveal the structural arrangement of the dust particles at a time during the simulation. This image was prepared by plotting the simulated particles in a 3-D coordinate system, with a sphere representing each particle. In this structure, each particle is in a cage defined by its nearest neighbors, but the structure is irregular, not crystalline. A video can be seen at [6] showing similar particles as shown in Fig. 1(a) from a rotating viewpoint.

To quantify the order of the 3-D structure, we calculate the pair correlation function $g(r)$ [7], [8]. For this liquid, $g(r)$ has only one distinctive peak in Fig. 1(b), indicating short-range translational order.

We characterize the dynamics using a wave spectrum. We start by using the particle position $\mathbf{r}_j(t)$ and velocity $\dot{\mathbf{r}}_j(t)$ to calculate the time series of the so-called longitudinal current, for a specified wave vector $\mathbf{k}$

$$J_L(k,t) = N^{-1} \sum_{j=1}^{N} [\dot{\mathbf{r}}_j(t) \cdot \mathbf{k}/|\mathbf{k}|] \exp[i\mathbf{r}_j(t) \cdot \mathbf{k}].$$

The spectral power $|J_L(k,\omega)|^2$ is then computed as the square modulus of the Fourier transformation in time of $J_L(k,t)$.

Results in Fig. 1(c) show that, as expected, the spectral power is concentrated along a curved band. The band has a great width in the ω-k space due to damping, arising from gas friction and the viscous motion of dust particles.

The band of spectral power in Fig. 1(c) corresponds to a real dispersion relation curve, which we plot in Fig. 1(c) as a dotted line. We determined this dispersion curve as the peak of the spectral power $|J_L(k,\omega)|^2$; to reduce the uncertainty, we computed ω for the peak as the first moment of the spectral power $|J_L(k,\omega)|^2$ for each value of k. The dispersion relation begins near $ka = 0$ with an acoustic-like upward slope, that is, the wave is forward for long wavelengths. This upward trend reverses for $2 < ka < 4$, where the dispersion relation curve has a slightly negative slope, that is, the wave is backward.

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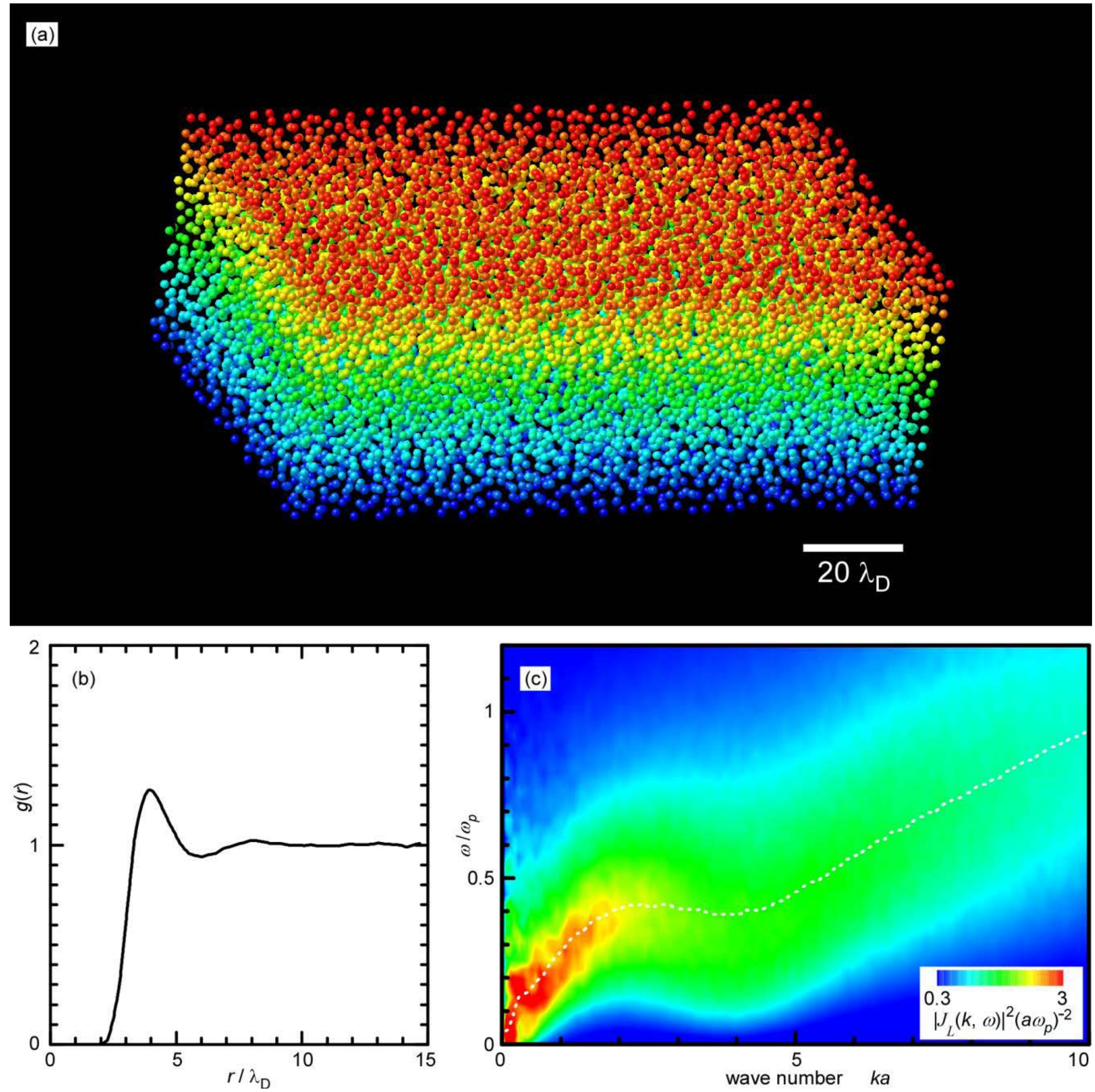

Fig. 1. (a) Structural arrangement of dust particles at a time during a 3-D Yukawa simulation. For clarity, the diameter of the dust particle microsphere is exaggerated here. Color indicates the height above the bottom plane. (b) Pair correlation function $g(r)$, indicating the disorder in the structural arrangement. (c) Wave spectrum for longitudinal motion, computed from particle positions and velocities. Color indicates spectral power, which is not smoothed; the results here have finite noise, which was reduced by averaging over eight runs of the simulation and 750 directions of **k**. Dotted curve: real dispersion relation, as obtained from the weighted peak of the spectral power with respect to frequency.

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