Single-particle Langevin model of particle temperature in dusty plasmas

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A model of heating a particle in a dusty plasma is developed to predict the particle kinetic temperature. A Langevin approach is developed, generalizing a familiar Brownian motion model. Particles are cooled by neutral gas while being heated by one of two mechanisms: fluctuating electric fields and randomly fluctuating particle charge. Expressions are derived for the particle temperature resulting from these mechanisms. In both cases, the balance of heating and cooling leads to a particle kinetic temperature that varies inversely with gas pressure. When a particle is electrostatically suspended against gravity, the temperature is independent of particle size when heated only by electric field fluctuations, whereas it increases with size when heated only by charge fluctuations. An experiment is reported to demonstrate the use of the model in analyzing laboratory data.

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I. INTRODUCTION

Laboratory dusty plasmas have recently proven to be useful in studying strongly coupled plasmas, plasma-dust interactions, and solid-liquid phase transitions. The dust particles are highly charged due to collection of plasma electrons and ions and strongly repel one another. Clouds of such particles are confined by electrostatic traps in several types of experimental systems [1–5]. Neutral gas drag damps the particle motions, allowing highly ordered structures called plasma crystals to form. The plasma crystals are easily studied experimentally since they are optically thin with particle sizes often >10 μm and interparticle spacings >100 μm. There have also been many theoretical studies of strongly coupled and dusty plasmas dealing with, for example, dust charging [6–10], levitation and confinement [11–13], interparticle potential energy [14–16], and phase transitions [16–20].

Anomalously high particle kinetic temperatures have been measured in several recent plasma crystal experiments [21–25]. These temperatures are often much higher than that of the neutral gas, to which the particles are collisionally coupled. Several analytic and computational models of this phenomenon have been presented previously [16,26–28], including an early review of the present model [29], but the precise mechanisms leading to these high temperatures are still unknown.

Here we present an analytic model of the kinetic particle temperature T based on a single-particle Langevin analysis. The Langevin approach has previously been used to model dusty plasmas [30,31]. In the first effort, Ref. [30], only Brownian heating by the neutral gas was considered in estimating T. That study omitted electrostatic heating, which is widely believed to be necessary to explain the observed temperatures, since they are often far higher than the neutral gas temperature [21–26,31]. Zhakhovskii *et al.* [31] included a particular kind of electrostatic heating, due to nonstochastic charge fluctuations.

Here we present a more general treatment of electrostatic heating, including heating by electric fields and random charge fluctuations. After developing the model, we show how the predicted temperatures scale with common experimental parameters (Sec. III), derive a simple equation for the temperature due to random charge fluctuations (Sec. IV), and demonstrate the use of the model in analyzing an experiment (Sec. V).

II. SINGLE-PARTICLE LANGEVIN MODEL

A model is developed using the single-particle Langevin equation of motion to predict a particle temperature T_L. This temperature is an estimate of the true particle kinetic temperature. This model neglects microscopic collective fluctuations in the plasma. Heating is due to a combination of Brownian interaction with the neutral gas and electrostatic fluctuations, while cooling is due to neutral gas drag. The calculation of T_L is performed in analogy with the standard Langevin treatment for the Brownian motion of a particle in a viscous medium.

The starting point for the calculation is the single-particle Langevin equation:

$$\frac{d^2x}{dt^2} = -\omega_0^2 x - \gamma \frac{dx}{dt} + \frac{1}{m}\xi(t). \tag{1}$$

Here, $x(t)$ is the coordinate of a single particle, m is the particle mass, $m\gamma v$ is the drag force, and $\xi(t)$ is the fluctuating part of the force acting on the particle. Equation (1) is the equation of motion for a driven, damped harmonic oscillator and our solution for T_L will be analogous to finding the average kinetic energy of such a system. The restoring force $-\omega_0^2 x$, valid for small particle displacements, is composed of a combination of the local interparticle repulsion and the global confining potential due to gravity and the dc electric field in the sheath. The interparticle contribution to ω_0 is likely to be at the dust plasma frequency ($\omega_p/2\pi \sim 10$ Hz). Nitter [13] has calculated ω_0 due to the vertical confining potential, finding frequencies of order 10 Hz. In addition, only the neutral gas drag [32,33] contribution to the

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drag force $m\gamma v$ will be considered, thus neglecting any dissipation due to interparticle collisions or particle-plasma interactions. Therefore, we take γ to be the neutral gas damping rate. We neglect ion drag and thermophoretic forces which we believe to have a negligible effect on the true particle temperature.

While the drag force cools the particle, the fluctuating force $\xi(t)$ acts to heat the particle. The balance of the heating and cooling power, due to $\xi(t)$ and $m\gamma v$, respectively, determines the true particle temperature.

To model the real many-particle system meaningfully, using only one particle with a single degree of freedom, we must be selective in choosing the kind of fluctuating force $\xi(t)$ to consider. In particular, considering that we want to find a temperature corresponding to random particle motions, only fluctuating forces that act differently on neighboring particles should be included. This means that forces that act uniformly on all the particles, or forces that have a wavelength long compared to the interparticle spacing, should not be included in $\xi(t)$.

Our strategy is to solve Eq. (1) for the mean-square velocity $\langle v^2\rangle$, which can then be used to compute the temperature, given by

$$T_L = m\langle v^2\rangle \tag{2}$$

for one degree of freedom. This will be done by the method of Fourier transforms.

First we review the relationship between mean-square quantities, correlation functions, and power spectra [34,35]. We define the Fourier transform pair for velocity as

$$v(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} v(\omega)e^{-i\omega t}d\omega,$$

$$v(\omega) = \int_{-\infty}^{\infty} v(t)e^{i\omega t}dt. \tag{3}$$

The velocity autocorrelation function is given by

$$C_v(\tau) = \langle v(t)v(t+\tau)\rangle = \lim_{\theta\to\infty}\frac{1}{\theta}\int_{-\theta/2}^{\theta/2} v(t)v(t+\tau)dt. \tag{4}$$

Here θ is the time interval for the integration, and the second equality holds with the assumption that $v(t)$ is stationary, having no preferred origin in time, and ergodic, so that $v(t)$ takes on all of its possible values for sufficiently long θ. The velocity power spectrum $G_v(\omega)$ and autocorrelation function are related by the Wiener-Khintchine relations:

$$C_v(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G_v(\omega)e^{-i\omega t}d\omega,$$

$$G_v(\omega) = \int_{-\infty}^{\infty} C_v(\tau)e^{i\omega\tau}d\tau. \tag{5}$$

Using $v(t)$ from Eq. (3) in the integral of Eq. (4) and taking the Fourier transform yields

$$G_v(\omega) = \lim_{\theta\to\infty}\frac{1}{\theta}|v(\omega)|^2. \tag{6}$$

We wish to derive an expression for $\langle v^2\rangle$. This can be written as the velocity autocorrelation function evaluated at $\tau=0$. Using Eq. (5), $\langle v^2\rangle$ can be expressed in terms of the power spectrum as

$$\langle v^2\rangle = \frac{1}{2\pi}\int_{-\infty}^{\infty} G_v(\omega)d\omega. \tag{7}$$

Now we solve Eq. (1) for the velocity, obtaining

$$v(\omega) = \chi(\omega)\xi(\omega), \tag{8}$$

where

$$\chi(\omega) = \frac{-i\omega}{m(\omega_0^2-\omega^2+i\gamma\omega)}$$

is the response function. Using Eq. (6), the velocity power spectrum is

$$G_v(\omega) = |\chi(\omega)|^2 G_\xi(\omega),$$

where

$$G_\xi(\omega) = \lim_{\theta\to\infty}\frac{1}{\theta}|\xi(\omega)|^2$$

is the power spectrum of the fluctuating force. Substituting this result into Eq. (7) yields the instantaneous mean-square velocity,

$$\langle v^2\rangle = \frac{1}{2\pi m^2}\int_{-\infty}^{\infty}\frac{\omega^2}{(\omega^2-\omega_0^2)^2+\gamma^2\omega^2}\,G_\xi(\omega)d\omega. \tag{9}$$

The fluctuating force has two parts, so that $\xi(t)=\xi_{Br}(t)+\xi_{ES}(t)$. The first, $\xi_{Br}(t)$, represents the random force on the dust grain due to collisions with neutral gas molecules. The second, $\xi_{ES}(t)$, represents the force due to electrostatic fluctuations in the plasma. It is expected that $\xi_{ES}(t)$ will have a random, turbulent part that acts similarly to the Brownian force and a part due to coherent wave motion. Note that the Brownian and electrostatic forces are uncorrelated, so that

$$G_\xi(\omega) = G_\xi^{Br}(\omega) + G_\xi^{ES}(\omega). \tag{10}$$

In the absence of electrostatic fluctuations ($\xi_{ES}=0$), the problem reduces to the usual treatment of Brownian motion [34,35]. The particle temperature T_{Br} for Brownian motion is obtained from Eq. (9) by assuming the spectrum is flat, i.e., $G_\xi^{Br}(\omega)$ is constant for a frequency ω ranging from 0 to well above ω_0. This yields

$$T_{Br} = \frac{G_\xi^{Br}(0)}{2m\gamma}. \tag{11}$$

The flat spectrum assumption is traditionally invoked because of the random, white noise nature of the Brownian force. With the additional assumption of thermal equilibrium, T_{Br} is equal to the neutral gas temperature and Eq. (11) is a statement of the fluctuation-dissipation theorem [35].

In analogy with the analysis for Brownian motion, we may now predict a temperature for a plasma crystal in the presence of electrostatic fluctuations. Using Eqs. (2), (9), and (10), the total particle temperature predicted by the Langevin model is

$$T_L = T_{Br} + T_{ES}, \tag{12}$$

where

$$T_{ES} = \frac{1}{2\pi m}\int_{-\infty}^{\infty}\frac{\omega^2}{(\omega^2-\omega_0^2)^2+\gamma^2\omega^2}\,G_\xi^{ES}(\omega)\,d\omega \tag{13}$$

is the contribution due to electrostatic fluctuations. The next step is to find an expression for the power spectrum of the fluctuating force $G_\xi^{ES}(\omega)$.

Here we list some general properties that must be satisfied by $\xi_{ES}(t)$ and its power spectrum $G_\xi^{ES}(\omega)$. First, as mentioned above, in the context of a real dusty plasma containing many particles, only short-wavelength fluctuations can move nearest-neighbor particles differentially and thus create incoherent particle motions. This rules out, for instance, modulations of the global plasma properties due to 60 Hz noise [25]. Second, only frequencies near ω_0 contribute significantly to T_L, since the factor in the denominator of Eq. (13) suppresses any contributions for $|\omega-\omega_0| \gg \gamma$. This will generally suppress the effect of any ion-acoustic turbulence for reasonable values of ω_0, for example. Third, the fluctuations must be present where the particles are located (which in the case of laboratory plasma crystal experiments is a region of strong dc electric field, such as a sheath or double layer).

Neglecting Brownian motion and ion drag, the forces acting on a particle are the electrostatic force due to the sheath electric field and the gravitational force, so that the total force is $F(t)=F_{ES}(t)+F_G$. The electrostatic force is given by $F_{ES}(t)=Q(t)E(t)$, where Q is the particle charge and E is the local electric field. In general, Q and E both fluctuate in time, so they can be expanded as $Q=Q_0+\delta Q(t)$ and $E=E_0+\delta E(t)$, where $Q_0\equiv\langle Q\rangle$ and $E_0\equiv\langle E\rangle$ are the average values. The force can now be written as

$$F(t)=F_0+\xi_{ES}(t),$$

where

$$F_0 \equiv Q_0E_0+F_G$$

and

$$\xi_{ES}(t)\equiv Q_0\,\delta E(t)+E_0\,\delta Q(t)+\delta Q(t)\,\delta E(t).$$

Equilibrium particle levitation requires that $F_0=0$, so that $F(t)=\xi_{ES}(t)$. Using this result and the expression above for $\xi_{ES}(t)$, the power spectrum of the electrostatic force fluctuations is given by

$$G_\xi^{ES}(\omega)=Q_0^2G_{\delta E}(\omega)+E_0^2G_{\delta Q}(\omega)+Q_0E_0G_{\delta Q\,\delta E}(\omega)+O(\delta^3), \tag{14}$$

where Eq. (6) has been used to write

$$G_{\delta E}(\omega)=\lim_{\theta\to\infty}\frac{1}{\theta}|\delta E(\omega)|^2,$$

$$G_{\delta Q}(\omega)=\lim_{\theta\to\infty}\frac{1}{\theta}|\delta Q(\omega)|^2,$$

and we define

$$G_{\delta Q\,\delta E}(\omega)=\lim_{\theta\to\infty}\frac{1}{\theta}[\delta Q^*(\omega)\,\delta E(\omega)+\delta Q(\omega)\,\delta E^*(\omega)].$$

These are the power spectra of the fluctuating electric fields and particle charge. Substituting Eq. (14) into Eq. (13) and rearranging terms yields

$$\begin{aligned} T_{ES} &= \frac{Q_0^2E_0^2}{2m\gamma}\int_{-\infty}^{\infty}\frac{\gamma\omega^2/\pi}{(\omega^2-\omega_0^2)^2+\gamma^2\omega^2} \\ &\quad\times\left(\frac{G_{\delta E}}{E_0^2}+\frac{G_{\delta Q}}{Q_0^2}+\frac{G_{\delta Q\,\delta E}}{Q_0E_0}+O(\delta^3)\right)d\omega \\ &\equiv T_{\delta E}+T_{\delta Q}+T_{\delta Q\,\delta E}+O(\delta^3). \end{aligned} \tag{15}$$

In the integrand of Eq. (15), the factor $(\gamma\omega^2/\pi)/[(\omega^2-\omega_0 2)^2+\gamma^2\omega^2]$ is peaked around $\omega=\pm\omega_0$ with width of order γ and is unit normalized. The physical consequence of this term is that only fluctuations with significant power at frequencies near ω_0 can efficiently heat the particles. A flat spectrum, analogous to the one assumed in deriving Eq. (11), is usually a reasonable approximation for a power spectrum G with a half width at half maximum (HWHM) σ satisfying $\sigma \gg \omega_0+\gamma$.

The first term of Eq. (15),

$$T_{\delta E}=\frac{Q_0^2E_0^2}{2m\gamma}\int_{-\infty}^{\infty}\frac{\gamma\omega^2/\pi}{(\omega^2-\omega_0^2)^2+\gamma^2\omega^2}\left(\frac{G_{\delta E}}{E_0^2}\right)d\omega, \tag{16}$$

represents the direct interactions of the local fluctuating electric field δE with the particle. No assumptions have been made about the nature or origin of δE, but recall that only fields that act differently on two neighboring particles can heat them. These include, for example, movement of or charge fluctuation on a neighboring particle, and plasma oscillations either generated in the particle layer or propagating from upstream. The contribution due to plasma oscillations at the sheath edge is considered below in Sec. V, where $T_{\delta E}$ is compared to experimental measurements of the temperature.

In some cases, $G_{\delta E}(\omega)$ is approximately flat out to some frequency $\sigma_E \gg \omega_0+\gamma$. This allows us to simplify Eq. (16). To do this, we first model the shape of $G_{\delta E}(\omega)$ as a Gaussian:

$$G_{\delta E}(\omega)=2\pi\langle\delta E^2\rangle\frac{1}{\sqrt{2\pi}\sigma_E}\exp\left(\frac{-\omega^2}{2\sigma_E^2}\right),$$

where the normalization is consistent with Eq. (7). Inserting this expression into Eq. (16) and using $\sigma_E \gg \omega_0 + \gamma$ yields the flat spectrum approximation,

$$T_{\delta E} \simeq \sqrt{\frac{\pi}{2}} \frac{Q_0^2 E_0^2}{m\gamma\sigma_E} \left(\frac{\langle \delta E^2 \rangle}{E_0^2} \right). \tag{17}$$

Note that the factor $1/\sigma_E$, which is equivalent to a correlation time for electric field fluctuations, indicates that broader spectrums have proportionally less power available to heat the particles at frequencies near ω_0.

The second term of Eq. (15),

$$T_{\delta Q} = \frac{Q_0^2 E_0^2}{2m\gamma} \int_{-\infty}^{\infty} \frac{\gamma\omega^2/\pi}{(\omega^2 - \omega_0^2)^2 + \gamma^2\omega^2} \left(\frac{G_{\delta Q}}{Q_0^2} \right) d\omega, \tag{18}$$

represents the interactions of the steady-state sheath electric field with charge fluctuations δQ on the particle. In a laboratory plasma crystal, this heating process would inject energy into vertical motion.

In cases where the spectrum $G_{\delta Q}(\omega)$ is flat up to a frequency $\sigma_Q \gg \omega_0 + \gamma$, Eq. (18) can be simplified as

$$T_{\delta Q} \simeq \frac{Q_0^2 E_0^2}{m\gamma\sigma_Q} \left(\frac{\langle \delta Q^2 \rangle}{Q_0^2} \right). \tag{19}$$

Here $G_{\delta Q}(\omega)$ has been assumed to have a Lorentzian shape with width σ_Q, as discussed below in Sec. IV.

Several different mechanisms can lead to charge fluctuations. One of these is the discrete nature of the charging process. Particles absorb individual electrons and ions from the plasma at random times, leading to random charge fluctuations [6–8]. To the extent that the fluctuations of charge on neighboring particles are uncorrelated, these fluctuations will lead to random, differential particle motions so that $T_{\delta Q}$ will predict a true temperature. This heating mechanism is investigated in Sec. IV below.

An entirely different contribution to δQ comes from the motion of a particle in a spatially inhomogeneous plasma potential, such as a plasma sheath or double layer [31]. This arises because the particle charge is dependent on local plasma conditions, so the charge will fluctuate as the particle moves about. The effect will be greatest, of course, when there are strong gradients in the plasma, with commensurately large dc electric fields. One example is the levitation of particles in a dc sheath or double layer [20,13]. Zhakhovskii *et al.* [31] proposed a heating model based on this effect. They pointed out that when both Q and E depend on position (in two or three dimensions), it is possible that the force on a particle is not derivable from a potential. Particles are thus able to move in a closed path and gain energy from the ambient dc electric field. In equilibrium this energy input is balanced by energy losses due to gas drag. It is not practical to model this effect with our method for two reasons. First, it intrinsically requires particle motion in at least two dimensions and second, it requires multiple interacting particles. Neither of these conditions is satisfied by our single-particle, one-dimensional model.

It has been suggested that another contribution to δQ could come from electrostatic fluctuations propagating from upstream of the particles [36]. These lead to fluctuations in the local plasma conditions, which in turn lead to charge fluctuations, much like the spatially inhomogeneous case discussed above.

The third term in Eq. (15),

$$T_{\delta Q\,\delta E} = \frac{Q_0^2 E_0^2}{2m\gamma} \int_{-\infty}^{\infty} \frac{\gamma\omega^2/\pi}{(\omega^2 - \omega_0^2)^2 + \gamma^2\omega^2} \left(\frac{G_{\delta Q\,\delta E}}{Q_0 E_0} \right) d\omega, \tag{20}$$

represents coupled effects between the particle charge and the electric field fluctuations. One such effect is Melandsø's collisionless damping [37], which is significant when the inverse particle charging time $1/\tau_{ch}$ is comparable to the dust plasma frequency ω_p. In most rf experiments, including the one discussed below, the electron and ion densities are high enough that $1/\tau_{ch} \gg \omega_p$, and Melandsø's damping can be neglected.

III. SCALING OF $T_{\delta E}$ AND $T_{\delta Q}$

We now discuss the scaling of $T_{\delta E}$ and $T_{\delta Q}$ with the usual dusty plasma parameters. The factor $Q_0^2 E_0^2/m\gamma$ in these equations can be rewritten as mg^2/γ by noting that $|Q_0 E_0| = mg$ for levitation equilibrium, where g is the acceleration due to gravity. Here we are assuming that the confinement is solely in the vertical direction.

When the neutral gas mean free path for collisions with the dust is long compared to the particle size, it is appropriate to use the Epstein drag force to compute γ [32,33]. In this case the damping rate is

$$\gamma = \delta \sqrt{\frac{8}{\pi}} \left(\frac{T_g}{m_g} \right)^{-1/2} \frac{P_g}{\rho a} = K_\gamma \frac{P_g}{\rho a} \ \ \mathrm{s}^{-1}, \tag{21}$$

where m_g, T_g, and P_g are the mass, temperature, and pressure of the neutral gas, a is the dust particle radius, and ρ is the individual particle mass density. The parameter δ is 1 for specular reflection or 1.39 for diffuse reflection of the neutral gas molecules from the dust particle [32]. We use $\delta = 1.39$, as suggested by Pieper and Goree's results in Ref. [23]. Note that the coefficient K_γ is inversely proportional to the neutral gas thermal velocity. This shows that the temperatures $T_{\delta E}$ and $T_{\delta Q}$ scale inversely with neutral gas pressure and $m_g^{1/2}$ (through K_g), as expected.

We can estimate mg^2/γ using typical experimental values for the various parameters. For krypton neutral gas at room temperature, $K_\gamma = 1.71\ \ \mathrm{mTorr}^{-1}\ \ \mu\mathrm{m\,g\,cm}^{-3}$. For the values typical of plasma crystal experiments ($a = 4.7\ \ \mu$m, $\rho = 1.5\ \ \mathrm{g/cm}^3$, and $P_g = 100$ mTorr) this yields $\gamma/2\pi = 3.86$ Hz and $mg^2/\gamma = 8.9$ MeV/s.

Further insight into the scaling of $T_{\delta E}$ can be obtained by evaluating the particle charge Q_0. This is essentially the charge on a spherical capacitor with an electric potential equal to the particle's floating potential. The capacitance is proportional to the particle radius a, while the floating potential is a multiple of T_e/e, where T_e is the electron temperature, so that the charge number $Z \equiv |Q_0/e|$ can be written as

$$Z = K_Q a T_e. \tag{22}$$

The numerical coefficient K_Q can be computed using the orbital-motion-limited (OML) charging model [38], as tabulated in the Appendix. For example, for krypton ions drifting at the ion acoustic speed and $T_e \gg T_i$, $K_Q = 2913\ \mu\text{m}^{-1}\ \text{eV}^{-1}$.

Using these results in Eq. (17) yields the desired scaling

$$T_{\delta E} \propto \frac{T_e^2}{P_g \sigma_E} \langle \delta E^2 \rangle. \tag{23}$$

It is particularly interesting that $T_{\delta E}$ is the same for large and small particles, since it is independent of a. This is true provided that the spectrum is flat up to a frequency $\sigma_E \gg \omega_0 + \gamma$. Having found the contribution to T_L due to electric field fluctuations, we will now use Eq. (19) to calculate the contribution due to random charge fluctuations.

IV. RANDOM CHARGE FLUCTUATION HEATING

In this section we derive an expression for $T_{\delta Q}$ when the charge fluctuations are due to the discrete, random nature of the charging process. Specifically, we will justify the form of Eq. (19), and then rewrite it in terms of more intuitive experimental parameters.

First we justify the flat spectrum approximation used to obtain Eq. (19). Cui and Goree [8] have shown that $G_{\delta Q}(\omega)$ has a significant low-frequency component due to the discrete nature of the electrons and ions that are collected by the particles. They reported that $G_{\delta Q}(\omega)$ is approximately Lorentzian in shape with a HWHM σ_Q proportional to the inverse of the charging time τ_{ch},

$$\sigma_Q = 0.024\left(\frac{2\pi}{\tau_{ch}}\right) = 0.15 K_\tau^{-1}\left(\frac{a n_e}{T_e^{1/2}}\right)\ \text{s}^{-1}, \tag{24}$$

where n_e is the electron density. The coefficient K_τ was found numerically using the method of Ref. [8] for use in comparing to the experiment (see Sec. V below). It is tabulated for a variety of plasma parameters in the appendix.

We are now prepared to examine the range of validity of the assumption that $\sigma_Q \gg \omega_0 + \gamma$, which justifies the form of Eq. (19). Considering the vertical confining potential, we first note that $\omega_0 \sim 3\gamma$ and like γ it appears to scale approximately as $1/a$ [13]. Since we do not have an analytic expression for ω_0, we write

$$\frac{\sigma_Q}{4\gamma} = 0.0375 (K_\gamma K_\tau)^{-1}\left(\frac{n_e a^2}{P_g T_e^{1/2}}\right)$$

using Eqs. (21) and (24) above. Using the previously given value for K_γ, $K_\tau = 4610\ \text{s}\ \mu\text{m}\ \text{m}^{-3}\ \text{eV}^{-1/2}$, a typical plasma density $n_e = 10^8\ \text{cm}^{-3}$, neutral pressure $P_g = 100$ mTorr, and $T_e = 2$ eV, we obtain $\sigma_Q/4\gamma \approx 3a^2$, where a is in μm. Thus the condition $\sigma_Q \gg \omega_0 + \gamma$ (where $\omega_0 + \gamma \approx 4\gamma$) holds, and Eq. (19) is valid, for $a \gtrsim 2\ \mu$m. The condition will also hold for smaller particles at higher plasma densities.

Cui and Goree also calculated the fractional mean-square charge fluctuations to be

$$\frac{\delta Q^2}{Q_0^2} = \frac{1}{4Z}, \tag{25}$$

a result verified analytically in Refs. [6] and [7]. Using Eqs. (19) and (25), we obtain

$$T_{\delta Q} = \frac{m g^2}{4\gamma \sigma_Q Z}. \tag{26}$$

From this and previously defined relations we get the scaling for the random charge fluctuation temperature,

$$T_{\delta Q} \propto \frac{\rho^2 a^2}{n_e T_e^{1/2} P_g}. \tag{27}$$

Note that random charge fluctuation heating is most significant for larger particles. This is mainly because larger particles must be levitated in a region of large dc electric field E_0, so that the random force $\delta Q E_0$ is commensurately large. Such a levitation force is not required if the particles are small, or if there is no gravity, as in recent microgravity experiments [39]. In that case, $T_{\delta Q}$ should be calculated from Eq. (18) rather than Eq. (26), and its magnitude will be significantly smaller than for large particles levitated within the sheath.

V. EXAMPLE EXPERIMENTAL APPLICATION

We now demonstrate how the models for predicting particle temperatures can be used with experimental data. The computation of $T_{\delta Q}$ using Eq. (26) requires the particle charge Q, which we determine from Langmuir probe measurements of T_e using the OML model, Eq. (22). The computation of $T_{\delta E}$ using Eq. (16) requires a spectrum of electrostatic fluctuations, as well as Q. In this experiment we use a Langmuir probe operated in ion saturation to measure the electrostatic fluctuations. The values computed for $T_{\delta Q}$ and $T_{\delta E}$ are compared with the temperatures computed from the directly imaged particle motions in the experiment.

A. Experiment

In the experiment, highly charged plastic microspheres were levitated by the strong electric field in the dc sheath above the powered electrode of a radiofrequency (rf) discharge plasma. The particles were $9.4 \pm 0.3\ \mu$m diameter with a mass density $\rho = 1.5\ \text{g cm}^{-3}$. After insertion into the plasma, several thousand particles were found to be arranged in a cloud of two to three vertically aligned layers above a capacitively coupled horizontal electrode. The particles had an interparticle spacing of $\Delta \approx 500\ \mu$m in the horizontal direction. The discharge conditions were varied by changing the krypton neutral gas pressure from 55 to 200 mTorr while fixing the peak-to-peak electrode voltage and driving frequency at 85 ± 2 V and 13.55 MHz, respectively.

In addition to levitating the particles, the dc electric field produces a substantial ion flow, originating in the main plasma and moving past the particles to the electrode. Thus, electrostatic fluctuations originating above the sheath edge may propagate with the ion flow to the particle layer. This is

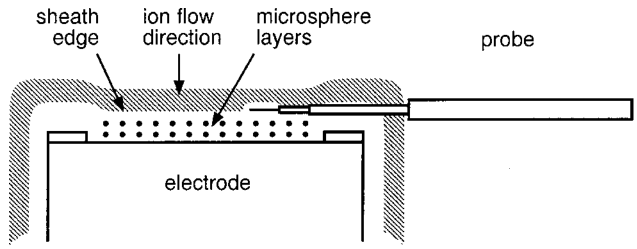

FIG. 1. Side view sketch of the electrode, particles, visible sheath edge, and Langmuir probe. The probe is used to measure ion density fluctuations upstream of the particles.

one possible source for the electrostatic fluctuations needed to compute Eq. (16).

We obtained the spectrum of low-frequency electrostatic fluctuations upstream of the particles in the following manner. A rf-compensated Langmuir probe was used to measure the ion saturation current, which is proportional to the ion density, just above the visible sheath edge. The probe location, shown in Fig. 1, was upstream of the particles, which were levitated within the sheath. To measure the ion saturation current, a bias voltage of -27 V was applied to the probe. The power spectrum of ion saturation current fluctuations $G_i(\omega)$ was computed from the fast Fourier transform of the ac portion of the voltage drop across a 1 MΩ resistor in the external probe circuit. This was normalized using the dc voltage across the same resistor. The result is the normalized power spectrum of ion density fluctuations,

$$G_i(\omega)=\lim_{\theta\to\infty}\frac{1}{\theta}\left|\frac{\delta n_i(\omega)}{n_i}\right|^2,$$

since the ion density n_i is proportional to the ion saturation current. In Sec. V C below we will write the spectrum of electrostatic fluctuations $G_{\delta E}(\omega)$, which is needed in Eq. (16), in terms of $G_i(\omega)$.

The particle charge, which is required in both Eq. (26) and Eq. (16), was determined from the experimentally measured electron temperature T_e using Eq. (22) and Table I of the Appendix (Kr, $T_e/T_i=80$, $U_i/C_s=1$). The same Langmuir probe described above was used to obtain the T_e measurements [40,41], which were ~ 4 eV and varied by about 10% over the measured pressure range.

Particle kinetic temperature measurements were also made, for comparison with the theoretically predicted temperatures. A vertical slice of the particle cloud was imaged using a long-distance microscope to obtain vertical and horizontal components of the particle velocities. Velocity distributions were obtained and were found to be approximately Maxwellian, with different temperatures for horizontal and vertical particle motions.

Because we positioned the Langmuir probe several millimeters upstream of the particle layer, our experimental fluctuation spectrum serves as a measure of fluctuations generated upstream. These propagate with the ion flow down toward the particle layer. It is possible, of course, that additional fluctuations may be generated in the particle layer, as proposed, for example, in Ref. [42]. Therefore, the present comparison of the temperatures measured in the experiment to the $T_{\delta E}$ predicted by the model will test whether fluctuations generated upstream of the particles accounts for their heating.

Further details of the apparatus and methods used in this experiment are presented in Ref. [25].

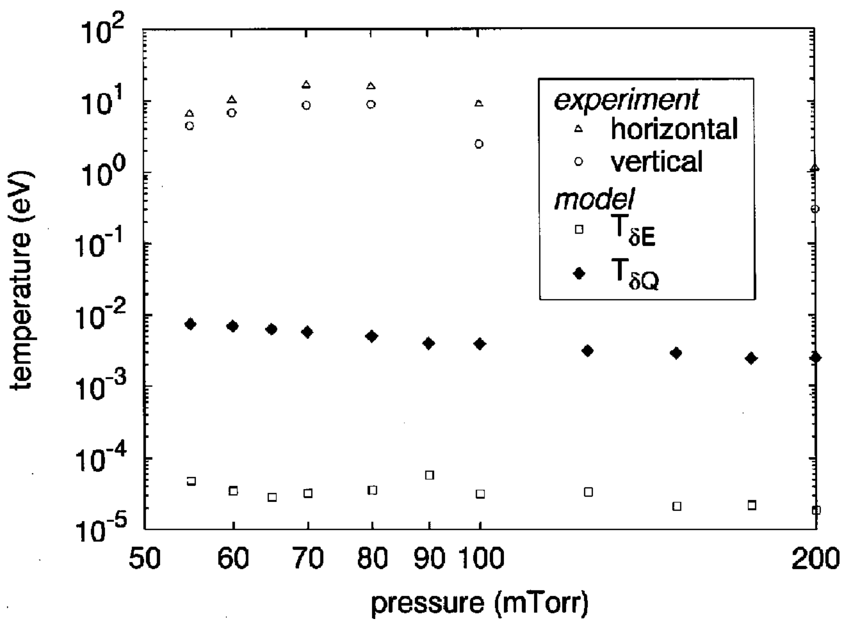

FIG. 2. Experimentally measured kinetic temperatures compared to the Langevin model predictions. Here, $T_{\delta E}$ predicts the temperature due to electrostatic fluctuations. The values shown were computed from Eq. (28) using fluctuation spectra measured experimentally upstream of the particle layer. The temperature due to random charge fluctuations, $T_{\delta Q}$, was computed using Eq. (26) and the experimentally measured value of T_e.

B. Comparison of experiment and Langevin model for particle charge

We now use our experimental results to compute the temperatures predicted by our model equations. We will then compare them to the directly measured temperatures.

First we consider $T_{\delta Q}$. Equation (26) above for computing $T_{\delta Q}$ requires the charge Q, which we determine from the experiment as described in the previous section, and the parameters σ_Q and γ. We computed σ_Q using Eq. (24) with K_τ taken from Table II in the appendix (Kr, $T_e/T_i = 80$, $U_i/C_s = 1$) and $n_e = 10^8$ cm^{-3}. The damping rate γ was calculated using the value for K_γ given in Eq. (21) above ($2\ \mathrm{Hz} \leqslant \gamma/2\pi \leqslant 8\ \mathrm{Hz}$ over the pressure range of this experiment). The charge number Z was calculated using the measured T_e data, as discussed above in Sec. V A.

The resulting predicted temperature $T_{\delta Q}$ due to random charge fluctuations is about two orders of magnitude smaller than the measured temperature. Figure 2 compares $T_{\delta Q}$ and the measured temperature over a range of pressures. The reason that $T_{\delta Q}$ is so small compared to the measured temperature is that a relatively small amount of the charge fluctuation power is available to heat the particles at frequencies $\omega \leqslant \gamma$. Note, however, that $T_{\delta Q}$ could be significant for larger particles, as indicated by Eq. (27).

C. Comparison of experiment and Langevin model for fluctuating electric fields

Now we consider $T_{\delta E}$. Equation (16) for computing $T_{\delta E}$ requires the electric field fluctuation spectrum, $G_{\delta E}(\omega)$, in addition to Q and γ. In the experiment we are able to measure the ion density fluctuation spectrum $G_i(\omega)$, as described in Sec. V A above. To rewrite $G_{\delta E}(\omega)$ in terms of $G_i(\omega)$, we combine the Boltzmann response equation, $\delta n_i/n_i = e\,\delta\phi/T_i$, and the relation $\delta E \approx \nabla(\delta\phi) \approx \delta\phi/\lambda$. Here $\delta\phi$ is the fluctuating part of the local potential, T_i is the ion temperature, and λ is the spatial scale for changes in $\delta\phi$. This allows us to rewrite Eq. (16) as

$$T_{\delta E} \approx \frac{T_i^2 Z^2}{m\gamma\lambda^2}\int_0^{\mathrm{BW}}\left(\frac{\gamma/\pi}{\omega^2+\gamma^2}\right)G_i(\omega)\,d\omega. \tag{28}$$

Here we have let $\omega_0 = 0$, thereby neglecting the restoring force due to the confining potential. It turns out that this approximation does not significantly alter the present results. Note that the integration limits in Eq. (28) have been changed from those of Eq. (16) to account for the fact that $G_i(\omega)$ is an experimentally measured, single-sided power spectrum.

We integrated Eq. (28) numerically from over a bandwidth of 1000 Hz, using the fluctuation spectrum $G_i(\omega)$ from the experiment and estimating the ion temperature $T_i \approx 300$ K. We assumed $\lambda \approx 250\ \mu$m, since those fluctuations with wavelengths shorter than Δ will be most effective in heating the particles. Values for the damping rate γ and charge number Z were the same as those used in computing $T_{\delta Q}$.

The resulting predicted temperature $T_{\delta E}$ is five orders of magnitude smaller than the measured temperature, as shown in Fig. 2. This is because the upstream electrostatic fluctuations $G_i(\omega)$ are weak, and because only a small fraction of their total power is contained in frequencies $\omega \leqslant \gamma$, where it can efficiently heat the particles. This discrepancy between $T_{\delta E}$ and the measured temperature provides evidence that fluctuations generated upstream of the particles are not responsible for heating them. This in turn suggests that fluctuations originating elsewhere, for example, in the particle layer itself, account for the heating.

VI. CONCLUSIONS

We have developed a simple model of particle heating in dusty plasmas. In addition to the Brownian interaction with the neutral gas, this model includes an electrostatic heating mechanism, which is needed in order to account for the large particle temperatures observed in many recent plasma crystal experiments. Two main types of electrostatic heating are discussed: heating due to the interaction of electric field fluctuations with the mean particle charge, and heating due to the interaction of charge fluctuations with a dc electric field.

Additionally, we have demonstrated how the input parameters of this model can be determined easily from experimental data. Specifically, predicting $T_{\delta Q}$ using Eq. (26) requires a measurement of the particle charge Q, while predicting $T_{\delta E}$ using Eq. (16) requires a spectrum of electrostatic fluctuations, in addition to Q. In our experiment, we determined Q from Langmuir probe measurements of T_e, using the OML model, Eq. (22). Note that other methods of measuring the charge could also be used, for example, the resonance method [43–45]. We measured the electrostatic fluctuations using a Langmuir probe operated in ion saturation.

Comparing the experiment to the model also led to the conclusion that in our experiment the fluctuations that account for the particle temperature must originate somewhere other than upstream of the particle layer, and that the random charge fluctuations also do not account for the observed temperature. These results suggest that fluctuations originating in the particle layer must be considered as likely candidates for the observed heating. However, charge fluctuations could represent an important heating mechanism for larger particles.

After this paper was submitted, the authors learned of another paper, Ref. [46], where a model for random charge fluctuation heating is developed using a Langevin approach.

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APPENDIX: ORBITAL-MOTION-LIMITED CHARGING THEORY

The orbital-motion-limited (OML) model is often used to compute the charge of a spherical particle in a plasma. The method relies on balancing the electron and ion currents. These currents depend on electron and ion densities and temperatures, as well as particle size and surface potential. For a negatively charged particle, electron currents are suppressed exponentially by the negative surface potential on the particle. The ion current also depends not only on T_i, but also

TABLE I. Values of the coefficient K_Q (μm^{-1} eV^{-1}) in Eq. (22). For each gas and T_e/T_i ratio, four cases are shown: nondrifting ions and ions drifting at one, two, and five times the ion acoustic speed.

		U_i/C_s			
Gas	T_e/T_i	0	1	2	5
He	1	2115	2156	2220	2121
	20	1497	2060	2256	2141
	40	1340	2063	2257	2142
	80	1187	2063	2258	2142
Ar	1	2772	2822	2913	2876
	20	2076	2700	2939	2894
	40	1901	2701	2940	2894
	80	1729	2703	2941	2894
Kr	1	2986	3040	3137	3118
	20	2271	2910	3161	3136
	40	2092	2912	3162	3136
	80	1915	2913	3163	3137

TABLE II. Values of the coefficient K_τ (s $\mu\text{m}\,\text{cm}^{-3}$ $\text{eV}^{-1/2}$) in Eq. (24). For each gas and T_e/T_i ratio, four cases are shown: nondrifting ions and ions drifting at one, two, and five times the ion acoustic speed.

		U_i/C_s			
Gas	T_e/T_i	0	1	2	5
He	20	1100	2030	2500	2320
	40	913	2030	2500	2320
	80	757	2030	2500	2320
Ar	20	2040	3780	4780	4640
	40	1698	3770	4780	4640
	80	1416	3780	4780	4640
Kr	20	2480	4610	5880	5800
	40	2070	4610	5880	5800
	80	1720	4610	5880	5800

on the speed U_i at which the ions flow past the particle. In our experiments the particles are embedded in the self-bias sheath so we expect $U_i/C_s > 1$, where C_s is the ion acoustic speed.

Using the OML model, the charge on a particle can be expressed as a function of radius a and electron temperature: $Q/e = K_Q a T_e$. Values of K_Q for various plasma parameters are listed in Table I. These were computed numerically by balancing the electron and ion currents, which were computed using Eq. (3.2) and Eq. (3.3) (nonflowing ions) or Eq. (4.4) (flowing ions) of Ref. [38], respectively. These are still only estimates of the particle charge since the model neglects the effect of plasma non-neutrality and rf fluctuations present inside the sheath [20].

The numerical solution of the OML model was also used to compute a particle charging time, which is used in computing the width of the charge fluctuation power spectrum [see Eq. (24) above]. We defined the charging time τ_{ch} as the time required for an initially uncharged particle to achieve a fraction $1-1/e$ of its equilibrium charge [8]. The equation $\tau_{ch} = K_\tau T_e^{1/2}/(a n_e)$ shows the dependence of the charging time on the plasma parameters. It also depends on the ratio U_i/C_s. The coefficient K_τ is tabulated in Table II for various plasma parameters.

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