Fluctuations of the Charge on a Dust Grain in a Plasma

Chunshi Cui and J. Goree, *Member, IEEE*

Abstract—**A dust grain in a plasma acquires an electric charge by collecting electron and ion currents. These currents consist of discrete charges, causing the charge to fluctuate around an equilibrium value $\langle Q \rangle$. Electrons and ions are collected at random intervals and in a random sequence, with probabilities that depend on the grain's potential. We developed a model for these probabilities and implemented it in a numerical simulation of the collection of individual ions and electrons, yielding a time series $Q(t)$ for the grain's charge. Electron emission from the grain is not included, although it could be added easily to our method. We obtained the power spectrum and the rms fluctuation level, as well as the distribution function of the charge. Most of the power in the spectrum lies at frequencies much lower than $1/\tau$, the inverse charging time. The rms fractional fluctuation level varies as $0.5\, |\langle N \rangle|^{-1/2}$, where $\langle N \rangle = \langle Q \rangle / e$ is the average number of electron charges on the grain. This inverse square-root scaling means that fluctuations are most important for small grains. We also show that very small grains can experience fluctuations to neutral and positive polarities, even in the absence of electron emission.**

I. Introduction

When a dust grain is exposed to a plasma, it acquires a charge by collecting electrons and ions, or through photo-electron or secondary-electron emission [1]. When emission processes are unimportant, the equilibrium charge will be negative because the flux of electrons to an uncharged surface is high compared to that of ions. On the other hand, when electron emission is significant, the equilibrium charge is positive.

In astrophysical and space plasmas, charged dust grains respond to electromagnetic fields, leading to rich physical phenomena [2]. For example, the grain's charge has been considered as a dominant factor in regulating the magnetic diffusion and ionization fraction in dense molecular clouds [3] and for grain motion in planetary magnetospheres [4]. In the laboratory, dust particulates can cause serious contamination problems in plasma discharges used for material processing [5]. In this case, the charge is, again, an essential issue in understanding the forces on these particulates in order to eliminate them from the processing device [6], [7].

Charging processes have been studied previously by many authors [8]. Most have used a "continuous charging model" which assumes that the charge on the grain is determined by currents collected from a plasma, and that these currents are continuous in time. This model, which neglects the discreteness of the charge carriers that make the currents, is reviewed briefly in Section II.

The charge on a grain can fluctuate, for two reasons. One cause is turbulence or other spatial and temporal variations in the surrounding plasma properties, especially in the electron temperature [2], [4]. The continuous charging model can be used to model this kind of charge fluctuation. This phenomenon can be significant when the plasma fluctuations are large and more rapid than the charging time of the grain, which may happen, for example, in some turbulent astrophysical plasmas.

In this paper we study another cause of charge fluctuations—the discrete nature of the charge carriers. Electrons and ions are absorbed at the grain surface at random times. For this reason, the charge fluctuates, even in a steady-state and uniform plasma. This idea has been reported previously in the literature. Morfill *et al.* [9] anticipated that the number of the charges residing on the grain, $N = Q/e$, fluctuates according to counting statistics with an rms fluctuation $\Delta N = |N|^{1/2}$. Choi and Kushner [10] developed a particle-in-cell simulation of the charging of a grain, yielding a time series for the charge that revealed fluctuations. Draine and Sutin [11] obtained the charge distribution function using a method that did not require finding a time series.

We report a detailed analysis of the fluctuations due to discrete charge carriers. The collection of discrete charges is handled by using the currents predicted by the continuous charging model to find the probabilities per unit time of particle collection. These probabilities depend on the grain potential, in the same way that the currents do. We use this scheme to develop a numerical simulation, as described in Section III. The amplitudes and time scales of the fluctuations of the discrete charges are evaluated for various conditions that are applicable to both astrophysical and laboratory dusty plasmas. In Section IV, these results are presented in a useful form. These results can also be used by the reader to determine, for a given problem of interest, which is the larger cause of charge fluctuations, discrete charge carriers or plasma fluctuations.

II. Continuous Charging Process

In this section, we review the familiar charging model where the discrete nature of the charge carriers is ignored. The currents collected by a grain are treated as if they are continuous in time. This brief review will be helpful in developing the discrete charging model in Section III.

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The authors are with the Department of Physics and Astronomy, University of Iowa, Iowa City, IA 52242.

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We assume an isolated spherical grain of radius a, immersed in a steady-state and uniform plasma with a shielding length λ. We will treat the case $a << \lambda << \lambda_{\rm mfp}$, where $\lambda_{\rm mfp}$ is a mean free path for ion or electron collisions. Most astrophysical and laboratory dusty plasmas satisfy this condition. We assume that the entire spherical surface is at an electric potential ϕ_s, measured with respect to the plasma potential.

Although the theory presented in this paper can be extended easily to include photo-electron and secondary-electron emission, these currents are not included here. This assumption is probably more applicable to laboratory plasmas, because electron emission can be important in space plasmas.

A. Orbital-Motion-Limited Currents

Electron and ion currents are collected by a spherical grain in the same way as by a spherical Langmuir probe. Under the condition $a \ll \lambda \ll \lambda_{\rm mfp}$, we can use the "orbital-motion-limited" (OML) probe theory [12], [13] to describe the current collection by the grain. For Maxwellian electrons and ions, characterized by temperatures T_e and T_i, the OML currents for a spherical grain are [1]:

$$\begin{array}{lcll} I_e & = & I_{oe}\exp(e\phi_s/kT_e) & \phi_s < 0 \\ I_e & = & I_{oe}(1+e\phi_s/kT_e) & \phi_s > 0 \\ I_i & = & I_{oi}\exp(-z_ie\phi_s/kT_e) & \phi_s > 0 \\ I_i & = & I_{oi}(1-z_ie\phi_s/kT_e) & \phi_s < 0 \end{array} \quad (1)$$

Here z_i is the electronic charge of the ions. The coefficients I_{oe} and I_{oi} represent the currents that are collected for $\phi_s = 0$. They are given by $I_{o\alpha} = n_\alpha q_\alpha (kT_\alpha/m_\alpha)^{1/2}\pi a^2 f_\alpha(\mathbf{w})$, where n_α is the number density of plasma species α and $f_\alpha(\mathbf{w})$ is a rather complicated function of the drift velocity $\mathbf{w}$ between the plasma and the grains [1]. If the drift velocity is much smaller than the thermal velocities, the coefficients can be simplified to:

$$I_{o\alpha} = 4\pi a^2 n_\alpha q_\alpha (kT_\alpha/2\pi m_\alpha)^{1/2}. \quad (2)$$

In (1) and (2), Maxwellian distributions are assumed, which is thought to be reasonable for most laboratory and space plasmas.

The charge Q is related to the surface potential ϕ_s by

$$Q = C\phi_s, \quad (3)$$

where C is the capacitance of the grain in the plasma. For a spherical grain satisfying $a \ll \lambda$, the capacitance is given by [1]

$$C = 4\pi\varepsilon_0 a. \quad (4)$$

Many dust grains are non-spherical, and so one should ask how the assumption of a spherical shape limits the validity of the theory. Since we are treating the case $a \ll \lambda$, the electrostatic equipotentials can be distorted from a spherical shape only within a radius $\ll \lambda$. Consequently, we believe that the spherical assumption introduces only a small error, as long as $a \ll \lambda$, as it is in many dusty plasmas.

B. Continuous Charging Model

A grain that is immersed in a plasma will gradually charge up, by collecting electron and ion currents, according to

$$dQ/dt = \Sigma I_\alpha. \quad (5)$$

The temporal evolution of the charge $Q(t)$ can be found by integrating (5), using suitable expressions for the currents I_α, such as (1) and (2). These currents depend on the surface potential, and therefore on the charge through (3). This means that (5) is of the form $dQ/dt = F(Q)$, where F is a non-linear function.

Note that the charge Q, the number of electronic charges $N = Q/e$ and the surface potential ϕ_s are all time-dependent quantities. Their time-averaged values are $\langle Q\rangle$, $\langle N\rangle$, and $\langle\psi_s\rangle$. The equilibrium potential is also called the floating potential, denoted by $\phi_f = \langle\phi_s\rangle$.

To find the equilibrium potential ϕ_f, one can set $dQ/dt = 0$ in (5), yielding

$$I_{oe}\exp(e\phi_f/kT_e) - I_{oi}(1 - e\phi_f/kT_i) = 0. \quad (6)$$

The solution has a form

$$\phi_f = K_\phi T_e \quad (7)$$

where $K_\phi = K_\phi(T_i/T_e, m_i/m_e)$ is a coefficient that must be determined numerically. Since electron emission is neglected here, and the electrons have a higher thermal velocity than ions, the floating potential and K_ϕ are both negative. Note that ϕ_f is independent of the grain's size, but it depends on the plasma temperatures. For example, a sphere in a hydrogen plasma with $T_e = T_i$ has the Spitzer [14] potential $\phi_f \approx -2.5kT_e/e$.

An expression for the equilibrium charge is found by combining (3), (4) and (7), yielding

$$\langle N\rangle = \langle Q\rangle/e = K_Q a T_e, \quad (8)$$

where $K_Q = K_Q(T_i/T_e, m_i/m_e)$ is also negative. Note that $\langle Q\rangle$ is proportional to both the grain size and the electron temperature but independent of the plasma density n.

Another quantity we shall need is the charging time τ, which indicates how fast a grain can charge up in a plasma. The literature contains several definitions of τ, and they all give the same result, except for small differences in a coefficient. One way is to define τ as the ratio of the equilibrium charge to the net current that the grain would collect when its charge is perturbed slightly from the equilibrium [15]. Another way is to define τ as an RC time, by making an analogy to a capacitor charging through a resistor [1]. We choose to define τ as the time required for an initially uncharged grain to approach its equilibrium charge within one e-fold, as sketched in Fig. 1. Using this definition, we found that the charging time can be expressed as

$$\tau = K_\tau \frac{T_e^{1/2}}{an}, \quad (9)$$

where $K_\tau = K_\tau(T_i/T_e, m_i/m_e)$. The fact that τ is inversely proportional to both a and n means that the fastest charging occurs for large grains and high plasma densities. In Table I we

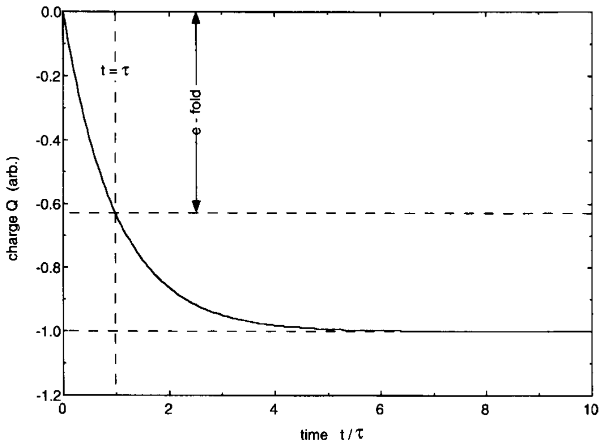

Fig. 1. Sketch of the temporal evolution of the charge on an isolated dust grain. The charging time τ is defined as the time required to approach the equilibrium within one e-fold, if the grain is initially uncharged.

TABLE I

COEFFICIENTS FOR ϕ, Q AND τ. THESE VALUES WERE FOUND BY A NUMERICAL SOLUTION OF THE CONTINUOUS CHARGING MODEL.

m_i (amu)	T_i/T_e	K_ϕ (V/eV)	K_Q (μm^{-1} eV^{-1})	K_τ (s μm cm^{-3} eV$^{-1/2}$)
1	0.05	−1.698	−1179	7.66×10^2
1	1	−2.501	−1737	1.51×10^3
40	0.05	−2.989	−2073	2.05×10^3
40	1	−3.952	−2631	3.29×10^3

list values for the three coefficients, K_ϕ, K_Q and K_τ, which we calculated for various plasma parameters.

III. DISCRETE CHARGING MODEL

In the preceding section, we reviewed the standard continuous charging model, which neglects the fact that the electron and ion currents collected by the grain actually consist of individual electrons and ions. We will now develop a charging model that includes the effect of discrete charges.

A. Physical Model

There are two key aspects of the collection of discrete plasma particles that we should identify and incorporate into our model. (We use the term "plasma particle" to refer to either electrons or ions.) First is that the time interval between the absorption of plasma particles varies randomly. Second is that the sequence in which electrons and ions arrive at the grain surface is random. But neither of these is purely random; they obey probabilities that depend on the grain potential ϕ_s.

Let us define $p_e(\phi_s)$ and $p_i(\phi_s)$ as the probability per unit time for absorbing an electron or ion, respectively. As the grain potential becomes more positive, more ions will be repelled and more electrons will be attracted to the grain, so p_i should decrease with ϕ_s and p_e should increase.

We calculate $p_e(\phi_s)$ and $p_i(\phi_s)$ from the OML currents $I_e(\phi_s)$ and $I_i(\phi_s)$,

$$p_\alpha = I_\alpha / q_\alpha. \tag{10}$$

This equation is the key to developing the discrete charging model. Basically, it converts the OML currents into probabilities per unit time of collecting particles. This relates the discrete charging model with its probabilities to the continuous charging model with its currents.

This assumption retains an important part, but not all, of the physics arising from the discreteness of the charge carriers. The discrete nature of an electron or ion is recognized when it is absorbed from the plasma by the grain, but not when it remains in the Debye sheath surrounding the grain. This is equivalent to treating the plasma with a continuum model that is characterized by the currents, I_α. This is a suitable approximation whenever the number of ions or electrons in a Debye sphere is much greater than unity. This condition is the usual plasma criterion, which is satisfied for almost all plasmas of interest. For this reason, we believe (10) adequately describes the discrete particle effects.

B. Numerical Simulation

In this section we describe a numerical simulation that implements the physical model presented above. As outlined in the flowchart in Fig. 2, the simulation starts with a zero charge, i.e., $Q_j = 0$ at $t_j = 0$. Then two steps will be repeated after each plasma particle is absorbed. In the first step, time is advanced by a random interval to the time when the next plasma particle j is absorbed. In the second step it is determined whether that particle was an electron or an ion, and the charge Q_j is updated accordingly. These two steps are explained in greater detail as follows.

1) First Step: Choose a Random Time Interval: In constructing a numerical simulation based on the physical model, we must retain the two key aspects of the randomness, as discussed in Section IIIA. The first requirement was that the plasma particles arrive at random time intervals, in a way that is consistent with probabilities that depend on ϕ_s. To handle this, the simulation will advance using a time step that is not fixed. Instead, there will be exactly one time step per particle that is collected. The time step will be chosen so that it corresponds to the time interval $\Delta t_j = t_j - t_{j-1}$ between the collection of plasma particles. This time step will be different every time, just the way it is in the physical system.

This random time step is chosen in a way that is consistent with the probabilities $p_e(\phi_s)$ and $p_i(\phi_s)$. It is not necessary to distinguish between electrons and ions; that is done in the second step of the simulation. The probability per unit time of collecting a plasma particle (whether ion or electron) is $p_{\text{tot}} = \Sigma p_\alpha$, where p_α is given by (10). The currents I_α depend on the grain surface potential ϕ_s, so p_{tot} also depends on ϕ_s and hence on the charge Q.

During a given time interval Δt, the probability of collecting a plasma particle is given by

$$P = 1 - \exp(-\Delta t p_{\text{tot}}). \tag{11}$$

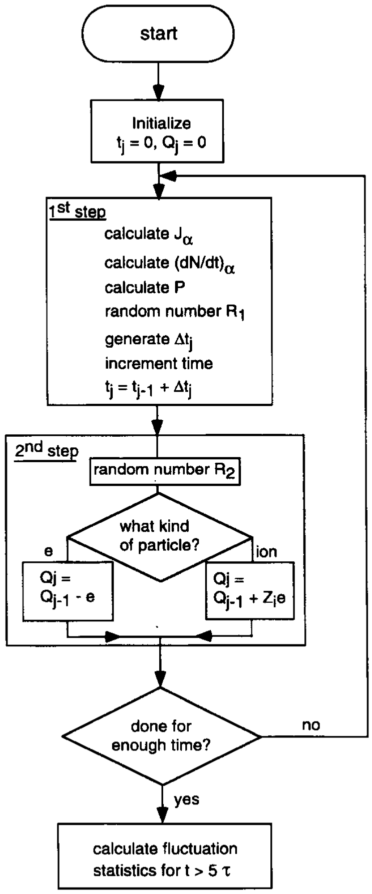

Fig. 2. Flow chart for the discrete charging model.

This is a probability that should not be confused with p_{tot} and p_α, which are probabilities per unit time. To calculate the random time interval, we generate a random number R_1, where $0 \leq R_1 \leq 1$, and equate it with the probability P in (11), yielding

$$\Delta t_j = -\frac{\ln(1 - R_1)}{p_{\text{tot}}}. \tag{12}$$

Before computing this, we must calculate the currents I_α from (1) and (2) to find p_α and then p_{tot}.

Note the physics that is retained in (12). The time interval Δ_t is random, because the expression contains a random number R_1. It also depends on the potential of the grain because of the ϕ_s dependence of p_{tot}. This is how the simulation retains the first key aspect of collecting discrete charges.

2) Second Step: Choose Electron or Ion: The second key aspect of the physical randomness was that plasma particles arrive in a random sequence, in a way that is consistent with probabilities that depend on ϕ_s. This requirement is handled in the second step of the simulation.

To determine whether the next absorbed particle is an electron or an ion, we again use a random number and the probabilities p_e and p_i. The probability that the next particle is species $\alpha(= e \text{ or } i)$ is simply p_α/p_{tot}. This is compared to a second random number R_2, where $0 < R_2 < 1$. If $R_2 < p_e/p_{\text{tot}}$, then the collected plasma particle was an electron, otherwise it was an ion.

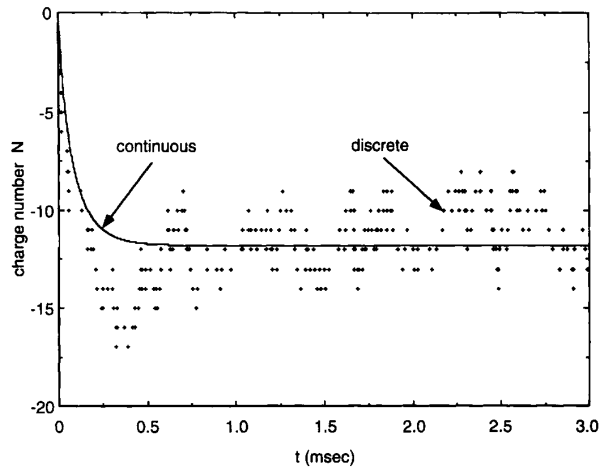

Fig. 3. Temporal evolution of charge number $N = Q/e$ for, $aT_e = 10$ nm eV, $T_i/T_e = 0.05$, $m_i = 1$ amu and $n = 10^{15}$ m^{-3}. Note that the discrete charging model reveals fluctuations about the smooth curve predicted by the continuous model.

We represent the charge by Q_j, which changes at time t_j when a plasma particle is collected. The charge is advanced by one increment, from $j-1$ to j, according to whether an electron or ion was absorbed

$$Q_j = Q_{j-1} - e \quad \text{if } R_2 < p_e/p_{\text{tot}} \text{ (electron absorption)}$$
$$Q_j = Q_{j-1} + z_i e \quad \text{if } R_2 > p_e/p_{\text{tot}} \text{ (ion absorption).}$$

This advances the charge and therefore ϕ_s. Then we must advance p_e and p_i before the next time step, because these probabilities per unit time depend on ϕ_s.

The simulation starts with an uncharged grain $Q = 0$ at $t = 0$. For good statistics it is allowed to continue for a time $\approx 100\tau$. It yields as its results the time series Q_j and t_j.

C. Simulation Results

Using the simulation described above, we calculated the time series for the charge. Fig. 3 presents results for a small grain, $aT_e = 10$ nm eV. For comparison, data is shown from both the discrete and continuous models. The continuous model gives a smooth curve for $Q(t)$. The discrete model shows how Q fluctuates in discrete steps about the smooth curve from the continuous model.

During the early transient period where the charge has not yet reached the equilibrium, the grain mainly collects electrons. This can be seen in the early part of Fig. 3, for $t < 0.25$ ms, when N changes usually by -1 rather than by $+1$. This happens because the grain potential is more positive than the equilibrium level, and therefore the probability for collecting electrons is much greater than for ions. Later, the grain's charge approaches the steady-state value $\langle Q \rangle$, and the

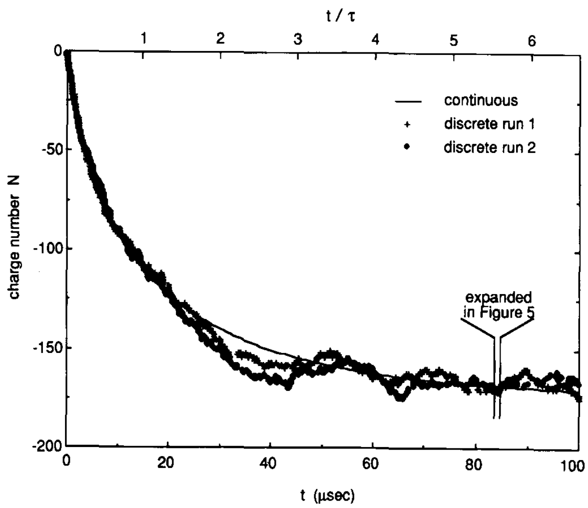

Fig. 4. Temporal evolution of charge number for a larger grain, with the same parameters as for Fig. 3, except $aT_e = 100$ nm eV. Two simulation runs for the discrete model are shown for different random number seeds.

probabilities p_e and p_i are less unequal. The fluctuations never disappear, however. The charge always fluctuates.

Fig. 4 shows the time series for a larger grain, with $aT_e = 100$ nm eV. Comparing the results for the small and large grains (Figs. 3 and 4 respectively) demonstrates that the larger grain charges more rapidly. Also note how the charge fluctuations are much less obvious for the large grain. This is an important result: larger grains have smaller fractional fluctuations. A more detailed quantitative analysis will be given in Section IV-C.

To show in detail the fluctuating charge, a portion of Fig. 4 is shown expanded in Fig. 5. One can see clearly that the charge changes in discrete steps $Q_j - Q_{j-1} = \pm e$, and that it changes at random time intervals.

IV. Fluctuations

In the previous section, we developed a simulation of the collection of discrete electrons and ions, and it revealed charge fluctuations. Here we characterize these fluctuations by several statistical measures: the power spectrum, charge distribution function, and rms fluctuation level. For these statistics, we only used the time series data for $t/\tau > 5$ to avoid the initial transient.

A. Power Spectrum

To reveal the nature of the fluctuations in the frequency domain, we computed the power spectrum $Q^*(\omega)Q(\omega)$. This was done using a fast-Fourier transform with a windowing algorithm to find $Q(\omega)$ from the time series $Q(t)$. Results are shown in Fig. 6, where the horizontal axis is the frequency $f = \omega/2\pi$, normalized by the inverse charging time τ^{-1}. This spectrum corresponds to the same time series whose beginning is shown in Fig. 4.

The power spectrum reveals that the fluctuations are dominated by very low frequencies. Half of the power in the

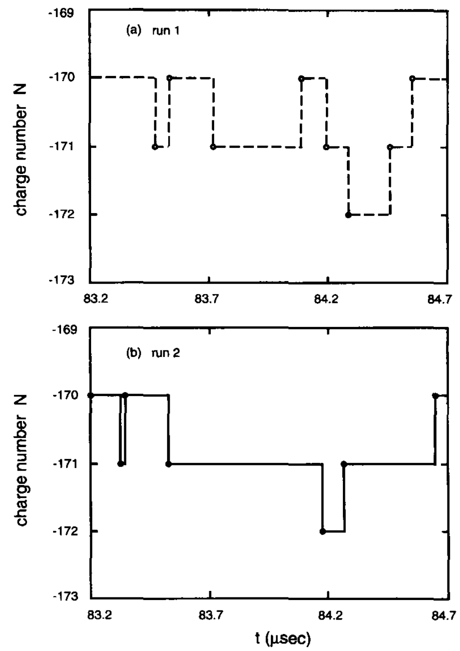

Fig. 5. The data of Fig. 4 are expanded to show a detail for $83.2 < t < 84.7$ μs. The two simulation runs are shown separately in (a) and (b). Note how the charge varies in discrete steps at random times, as individual electrons and ions are collected.

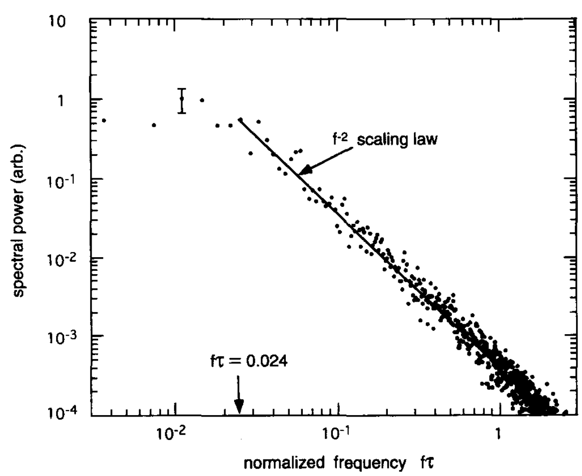

Fig. 6. Power spectrum of charge fluctuations. The solid line shows the power law f^{-2}. Most of the power lies below $f\tau = 0.024$ which is marked by the arrow. This spectrum was prepared from the time series of Fig. 4, except that only data from $5 < t/\tau < 105$ was included.

spectrum lies below $f\tau = 0.024$. This means that the fluctuations are mainly rather slow, compared to charging time of the initial transient. This is one of the main results of this paper. At higher frequencies, $f\tau > 0.024$, the spectral power diminishes as the second power of frequency, f^{-2}.

B. Charge Distribution Function

In dusty plasma theories one is often interested in a plasma containing many grains. Since each grain's charge fluctuates, it

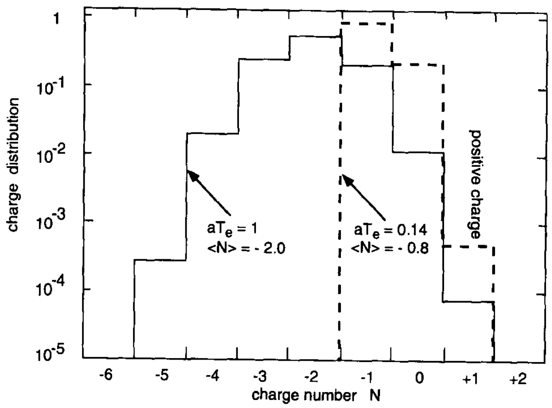

Fig. 7. Charge distribution functions. Data is shown for small grains in a hydrogen plasma with $T_i/T_e = 1$. The average charge is $< N >= -0.8$ and -2.0 for $aT_e = 0.14$ and 1 nm eV, respectively. Note that the charge sometimes fluctuates to a positive polarity.

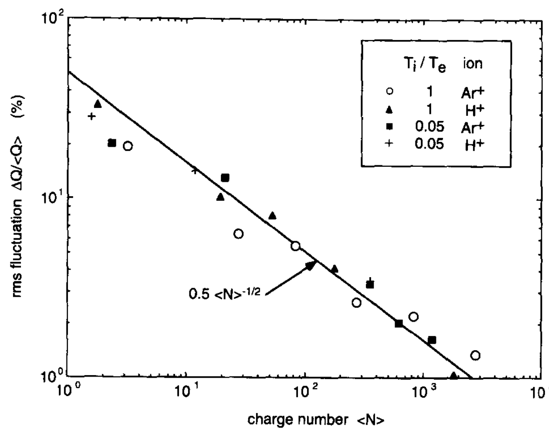

Fig. 8. Fractional rms fluctuation as a function of charge number. The four sets of plasma conditions are the same as listed in Table I. The representative error bar is due to the finite simulation time.

would be desirable to have a distribution function that indicates the fraction of the grains having a certain charge at a given time. We can arrive at such a distribution function from our data. Since our simulation assumes an isolated grain, our result is valid only for the limiting case where the grain density is low enough that each grain is effectively isolated from the others.

1) Histogram: We determined the charge distributions from the time series by making a histogram of the time spent at each charge level. Fig. 7 shows distribution functions for two cases with small grains, $aT_e = 0.14$ nm eV and 1 nm eV. Both charge distributions have peaks that are centered near the average charge $\langle N \rangle$. The distribution function is wider for larger aT_e. Our distributions can be compared to those of Draine and Sutin [11], who assumed electron and ion currents that were different from the OML currents we used.

The charge distributions shown in Fig. 7 differ from a Poisson distribution. Poisson statistics are suitable for describing random events where the probability for one event does not depend on the history of previous events. This does not apply to our problem, where the event is the absorption of an ion or an electron by a grain in a plasma. The rate of absorbing ions and electrons depends on the grain's charge, and therefore on the history of the absorption. After a grain collects an electron, for example, the probability of absorbing another electron is reduced, because the grain has charged negatively and tends to repel electrons more. In the discrete charging model, this physics appears in the dependence upon ϕ_s of the probabilities per unit time p_α.

2) Grains with a Non-Negative Charge: One might ask whether the charge can fluctuate to zero or even to a positive polarity. While it is well known that the grain can have a positive value when electron emission is present, it may be surprising to learn that it can also happen, momentarily, in the absence of emission.

For small grains, the charge fluctuates to non-negative polarities, as shown in Fig. 7. For this to happen, aT_e must be small enough that the $\langle Q \rangle$ is only a few electron charges. Even then, only a small fraction of grains have a positive charge. For both cases shown in Fig. 7, the probability is less than 10^{-3} that a grain has a positive charge of N = +1. So it is possible for grains to fluctuate to positive polarities in the absence electron emission, but this happens infrequently. Of course, fluctuations to positive values would be dramatically increased if there is electron emission.

Fluctuations lead to a neutral charge more often than to a positive charge. For the smallest grain shown in Fig. 7, $aT_e = 0.14$ nm eV, the grain has a neutral charge 20.3% of the time. One consequence of this is that small grains may be able to collide with one another and coagulate more often than one would expect from calculating the repulsion corresponding to the time-averaged charge.

C. RMS Fluctuation Level

As a final statistical measure of the fluctuations, we calculate their amplitude, expressed as the root-mean-square (rms) level. This was done using the time series Q_j from the discrete charging model. We normalized this by the equilibrium charge to yield the fractional rms fluctuation level, $\Delta Q/\langle Q \rangle = \langle(\langle Q \rangle - Q_j)^2\rangle^{1/2}/\langle Q \rangle$. Varying the grain radius from 1 nm to 1μm allowed us to vary $\langle$Q$\rangle$, which is the horizontal axis in Fig. 8.

The fluctuation level results in Fig. 8 reveal a simple power-law relation,

$$\Delta Q/\langle Q \rangle = 0.5|\langle N \rangle|^{-1/2}, \tag{13}$$

which is accurate for a wide range of plasma and grain parameters, provided that $|\langle N \rangle| > 1$. This is one of the chief results of this paper: the fractional rms fluctuation level is inversely proportional to the square root of the charge number on the grain.

Note that the $N^{-1/2}$ scaling means that smaller grains have larger fractional fluctuation levels. For a tiny dust grain with an average charge of only a few electrons, the fluctuations are enormous. On the other hand, for a very large object in a plasma, such as a Langmuir probe or a spacecraft, the fluctuations are quite negligible.

As predicted by Morfill *et al.* [9], this scaling is the same as in counting statistics, where the uncertainty of a count N is $N^{1/2}$, and the fractional uncertainty is $N^{-1/2}$. However, the coefficient in (13) cannot be predicted intuitively or by directly applying Poisson statistics. It depends on how the probabilities vary with the surface potential ϕ_s which comes from non-trivial ϕ_s dependence of the currents in (1) and (5). We found that this coefficient is 0.5 by using our simulation.

D. Applications

The idea that a grain's charge will fluctuate has implications for both astrophysical and laboratory dusty plasmas. It is possible that these implications have not been reported heretofore.

In the area of space plasmas, there are several effects. For one, the fluctuations of the charge Q on the grain can alter the dust motion in a plasma. The orbits become more irregular because both the electrostatic force QE and the Lorentz force $Qv \times B$ will fluctuate as Q fluctuates, even if the electromagnetic fields are constant. Secondly, photo-electron emission is often an important process in space plasmas, which can lead to the charge fluctuating between positive and negative values (see Section IV-B). This would tend to promote grain growth by coagulation, because oppositely-charged grains would attract one another. Moreover, fluctuations might alter the dispersion relations [2] and scattering [16] of waves in a plasma, due to the dependence on Q, which has a distribution function, even for mono-dispersive particulates. This latter effect may not be as important as a dispersion in grain sizes, however, if the dispersion is large.

For plasma processing experiments, it may be useful to know that the grain's charge can fluctuate between positive and negative values if there is enough electron emission. One might plan a process to heat the electrons to promote secondary emission or use an ultraviolet lamp for photo-electron emission. This could lead to a Q that is positive at least some of the time due to fluctuations, if not all of the time. A positive charge is important because it is believed that only negatively charged grains are confined in a laboratory discharge. Contaminants might be expelled from the plasma by promoting emission from the grains.

One type of basic physics experiment that has been reported is an effort to form a crystalline lattice of charged grains suspended in a plasma [17], [18]. Such a lattice would develop in a phase transition when the dust temperature is low enough. The charge fluctuations described in this paper lead to a fluctuating inter-grain potential that would have an effect similar to the random thermal motion. For this reason, crystal formation might be inhibited to a degree by charge fluctuations. This suggests using larger grains to avoid the problem.

V. Summary

We have studied the fluctuations of the charge of a spherical dust grain in a low-temperature plasma. The fluctuations around an equilibrium value $\langle Q \rangle$ arise from collecting discrete charges at random intervals.

We compared simulations of the charge using two models, the continuous and discrete charging models. The discrete model should be the more accurate of the two. Both models were based on the assumption $a \ll \lambda \ll \lambda_{\rm mfp}$, which allows the use of "orbit-limited" probe theory for current collection.

We used the discrete charging simulation to characterize the charge fluctuations, and we found the following useful results. The power spectrum of the fluctuations is concentrated at low frequencies, $f < 0.024\tau^{-1}$, and at higher frequencies the spectral power diminishes as f^{-2}. The distribution function for Q was found to be peaked about the equilibrium value $\langle N \rangle$, and for small grains it extends to positive polarities, even in the absence of electron emission. The rms fluctuation level varies as $0.5|\langle N \rangle|^{-1/2}$. It is valuable to note from the latter result that larger grains have smaller fractional fluctuation levels.

In our models we neglected any electron emission from the grains, although it would be straight-forward to add emission currents to our approach. Emission tends to make the grain's charge positive, or at least less negative. One important fact to know about electron emission is that it could cause the charge to fluctuate often between positive and negative values.

As we stated in the Introduction, charge fluctuations can happen not only due to the discrete charging effects studied in this paper, but also due to fluctuations in the plasma parameters. Fluctuations in electron temperature that are faster than the charging time, for example, will lead to a fluctuating grain charge. For any problem of interest, the reader can determine which cause of fluctuations is more significant by comparing their amplitudes and time scales. Our results, using (13) for the amplitude and Fig. 6 for the time scale, can be used in making such a comparison.

After submitting this paper, the authors learned of two other theoretical studies of charge fluctuations in a dusty plasma. Tsytovitch [15] treated the problem of a plasma with an ensemble of grains. He found that the fluctuation amplitude increases with dust number density, because a fluctuation on one grain causes a change in the current collected by another grain. In the limit of zero dust density, his approach yields a zero fluctuation level, which does not match our result (13) for the isolated grain. Jana *et al.* [19] investigated collective effects, such as unstable ion waves, that are brought about by charge fluctuations. The latter fluctuations are coherent, unlike our random fluctuations.

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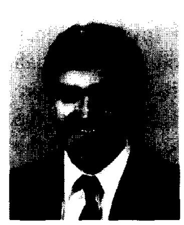
Chunshi Cui received the B.S. in Modern Physics from the University of Science and Technology of China in 1983. She received the Ph.D. in Plasma Physics from the Australian National University in 1992. Her Ph.D. thesis topic was an experimental study and modeling of a low-pressure RF generated magnetoplasma.

Since April 1993, she has been working on dusty plasmas as a research investigator.

Dr. Cui is a member of the American Physical Society.



John A. Goree received the B.S. in Applied Physics from the California Institute of Technology in 1980. He received the M.S. and Ph.D. degrees in 1982 and 1985 from the Plasma Physics Section of the Division of Astrophysical Sciences at Princeton University.

In 1985 he joined the faculty of the Department of Physics and Astronomy at the University of Iowa, where he is now Associate Professor. In 1991 and 1992 he was a guest scientist at the Max-Planck-Institut für extraterrestrische Physik in Garching, Germany. He has published over thirty refereed papers reporting experiments and theories for fusion plasmas, waves in plasmas, sputtering magnetron discharges, sheaths, laser diagnostics of plasmas, dusty plasmas, and scientific instrumentation.

Prof. Goree is a member of the American Geophysical Society, American Physical Society, and the American Vacuum Society. He has also served on the Executive Committee and Program Committees for the AVS Plasma Science and Technology Division.