Double lock-in detection for recovering weak coherent radio frequency signals

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(Received 11 February 1985; accepted for publication 25 April 1985)

A single radio frequency lock-in amplifier reduces broadband noise, but not rf pickup of the same frequency as the signal. If this pickup noise is at least 14 dB stronger than broadband noise, after both have passed through the lock-in, then the signal-to-noise ratio can be improved by applying the lock-in output to a second, low-frequency lock-in which is synchronized to an independent modulation of the signal. Weak coherent radio frequency signals buried in both rf pickup and broadband noise can be recovered by using this double lock-in method, as demonstrated in a plasma diagnostics experiment.

I. THEORY

Lock-in amplifiers are instruments for recovering, by bandwidth narrowing, a signal buried in broadband noise.[1] Recently, lock-in amplifiers have been introduced commercially for rf use; e.g., the EG&G PARC model 5202 operates up to 50 MHz, and the user can downmix signals of higher frequencies. Since rf pickup at these frequencies is much more severe than at 100 kHz (the highest frequency for most lock-in models), pickup may surpass broadband noise as the most difficult problem in signal recovery. If it is the same frequency as the signal, it will pass unattenuated through the lock-in, just as the signal does, and render lock-in detection useless. For use in harsh rf pickup environments, where shielding and other passive methods are insufficient, a technique called the "double lock-in method" actively reduces pickup noise. The theory of this simple method and the derivation of an expression for the signal-to-noise ratio (SNR) are described below. A practical criterion is presented for choosing between the single and double lock-in methods.

A. Single lock-in

A single lock-in amplifier, as shown in Fig. 1, reduces broadband noise by bandwidth narrowing.[1] The rms noise voltage is reduced by a factor proportional to $(\tau_1 f_1)^{1/2}$, where f_1 is the frequency of the rf signal detected, and τ_1 is the time constant of the lock-in (τ_1 equals the RC time of a low-pass filter in the output stage of the instrument). If amplitudes are scaled so that the contribution of broadband noise to the output power of the lock-in is $V_{BB}^2/(\tau_1 f_1)$, then the output signal is

$$V_{\rm OUT}^{\rm SLI} = \left(S^2 + \frac{V_{BB}^2}{\tau_1 f_1} + V_{\rm rfp}^2\right)^{1/2}, \tag{1}$$

where S is the true signal and $V_{\rm rfp}$ is the result of pickup.

The phase and amplitudes of the rf pickup fluctuate with time in a lab environment. No general model of the fluctuations may be predicted to describe every situation. A dc level of rf pickup amplitude is not as harmful, if it is weak, since it will appear only as a removable offset in $V_{\rm OUT}^{\rm SLI}$. The most undesirable measurements have slow pickup fluctuations that may be partly due to movements of people, cables, and instruments in the lab, and these deteriorate the quality of $V_{\rm OUT}^{\rm SLI}$.

The SNR for the single lock-in is

$$\mathrm{SNR}^{\rm SLI} = \frac{S}{(V_{BB}^2/\tau_1 f_1 + V_{\rm RFP}^2)^{1/2}}. \tag{2}$$

B. Double lock-in

The method uses a second, ordinary low-frequency lock-in after the first rf lock-in, as shown in Fig. 2. The key feature is that low-frequency modulation is applied to the rf signal but not to the rf pickup. For example, in optical detection, where amplitude modulated light carries the rf signal, a mechanical chopper in front of the optical detector can modulate the rf signal without affecting the mechanism of rf pickup (capacitive coupling of nearby powerful rf transmission lines and antennae to the detector, cable, and preamplifier). The second lock-in is synchronized to the chopper modulation and thus reduces the unmodulated rf pickup contribution to the output of the first lock-in. It also further reduces broadband noise, since that noise is unsynchronized to the modulation.

The second lock-in reduces $V_{\rm rfp}$ and the remaining effects of broadband noise by a factor $(\tau_2 f_2)^{1/2}$, where τ_2 is the time constant of the second lock-in and f_2 is the modulation frequency. The frequency ordering is

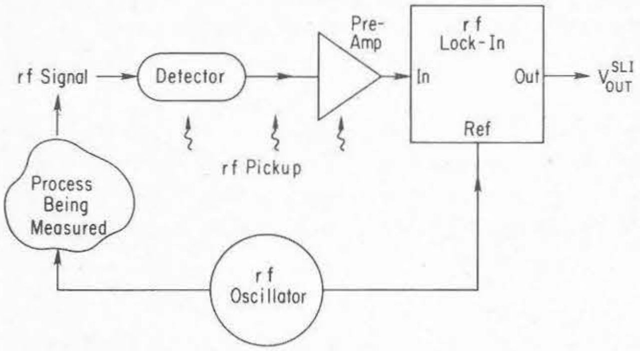

FIG. 1. Single lock-in detection. An rf oscillator produces a signal of frequency f_1 which is detected.

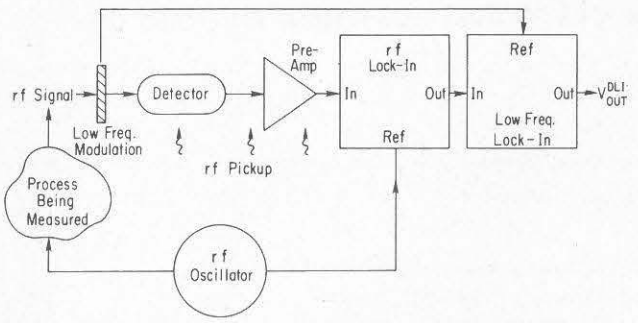

FIG. 2. Double lock-in detection. A second, low frequency lock-in, synchronized to a modulation at frequency f_2, is added to the single lock-in configuration. The signal is modulated before it reaches the detector, and the second lock-in is synchronized to this modulation in order to eliminate the noise due to rf pickup passing through the first lock-in.

$$\tau_2^{-1} \ll (2\pi\tau_1)^{-1} \simeq f_2 \ll f_1. \tag{3}$$

The time τ_2 should be as long as the time scale of the slowest fluctuations in $V_{\rm rfp}$ that might hinder one's measurements.

There is a price that is paid for the advantage of reduced rf pickup in the double lock-in method: the second lock-in diminishes the signal in two ways. First, the detected power is halved by modulating it. Second, the output stage of the first lock-in has a low-pass filter which attenuates the modulation by the ratio $V_{\rm out}/V_{\rm in} = F(\alpha)$, where F is the frequency response function of the low-pass filter and

$$\alpha \equiv 2\pi f_2 \tau_1 \tag{4}$$

is the dimensionless time constant of the filter. Thus the signal is reduced by $F(\alpha)/\sqrt{2}$. The final output of the double lock-in is

$$V_{\rm OUT}^{\rm DLI} = \left(\frac{[F(\alpha)]^2}{2} S^2 + \frac{V_{BB}^2}{\tau_1 f_1 \tau_2 f_2} + \frac{V_{\rm rfp}^2}{\tau_2 f_2}\right)^{1/2} \tag{5}$$

and the SNR is

$$\mathrm{SNR}^{\rm DLI} = \frac{F(\alpha)}{\sqrt{2}} \frac{S}{(V_{BB}^2/\tau f_1 \tau_2 f_2 + V_{\rm rfp}^2/\tau_2 f_2)^{1/2}}. \tag{6}$$

The user surely desires the maximum possible SNR and has available three variables to adjust for that purpose: the time constant τ_2 of the final lock-in, the time constant τ_1 of the first lock-in, and the modulation frequency f_2. The following choices minimize the last term in Eq. (5), the rf pickup: τ_2 should be as long as possible without losing the desired time response of the output signal, and τ_1 should be as short as possible in order to allow a high modulation frequency f_2. The variables τ_1 and f_2 are related through Eq. (4) so that $\mathrm{SNR}^{\rm DLI} \propto F(\alpha)\sqrt{\alpha}$. For the case $F(\alpha) = (1+\alpha^2)^{-1/2}$, the frequency response of a simple one-pole 6-dB/octave RC low-pass filter,[1] which is the kind used in commercial lock-ins, the SNR is maximized when $\alpha = 1$. Having chosen τ_1, the optimum modulation frequency is

$$f_2 = (2\pi\tau_1)^{-1}.$$

To find when the additional noise reduction of the double lock-in method outweighs its loss of signal, the SNR's of the double and single lock-ins are compared through the figure of merit

$$Q \equiv \frac{\mathrm{SNR}^{\rm DLI}}{\mathrm{SNR}^{\rm SLI}}.$$

Using Eq. (2) and Eq. (6),

$$Q = \frac{F(\alpha)}{\sqrt{2}} \frac{(V_{BB}^2/\tau_2 f_1 + V_{\rm rfp}^2)^{1/2}}{(V_{BB}^2/\tau_1 f_1 \tau_2 f_2 + V_{\rm rfp}^2/\tau_2 f_2)^{1/2}}, \tag{7}$$

where the same final time constant τ_2 is used for both methods to be fair in the comparison. By turning the source of rf pickup on and off while blocking the signal so that only noise is observed, the user of the single lock-in can easily compare $V_{\rm rfp}^2$ to $V_{BB}^2/(\tau_2 f_1)$. When the ratio R^2 is above a critical value, i.e., when rf pickup reaches a certain level, the SNR benefits from using the double lock-in. The critical value is derived below.

Using

$$R \equiv \frac{V_{\rm rfp}}{V_{BB}/(\tau_2 f_1)^{1/2}},$$

Eq. (7) for the figure of merit becomes

$$Q = \frac{F(\alpha)}{\sqrt{2}} \frac{(1+R^2)^{1/2}}{(1/\tau_1 f_2 + R^2/\tau_2 f_2)^{1/2}}$$

and choosing the optimum $\alpha = 1$,

$$Q = \frac{1}{2} \frac{(1+R^2)^{1/2}}{(2\pi + R^2/\tau_2 f_2)^{1/2}}.$$

Since the critical value of R should be small compared to the large number $\sqrt{2\pi\tau_2 f_2}$, one can approximate

$$Q \simeq \frac{(1+R^2)^{1/2}}{2\sqrt{2\pi}}. \tag{8}$$

The double lock-in method is desirable when $Q > 1$. Equation (8) shows that this is the case when $R > 4.9$.

The single lock-in method is preferable as long as the pickup contribution to its output is no more than 4.9 times as large (i.e., no more than 14 dB larger) as the broadband noise contribution. Measurements of weak signals in a severe pickup environment may not meet this test for using the single lock-in. If the user blocks the signal in order to examine the noise level, and finds that turning off the source of rf pickup lessens the noise level at the lock-in output by a

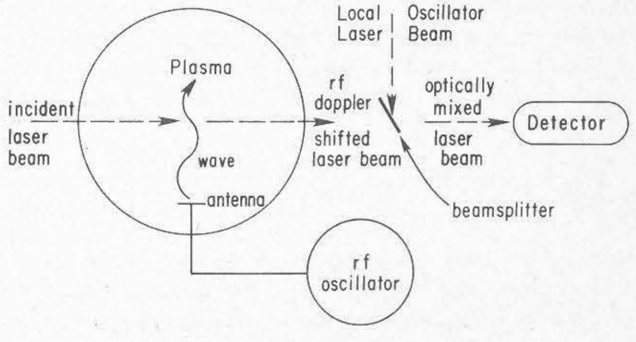

FIG. 3. Schematic of demonstration experiment for double lock-in technique. A weak rf signal is produced by the detector by optically mixing two far-infrared laser beams, one of which has been Doppler shifted in frequency by an rf wave in the plasma. A rotating mechanical chopper modulates the signal.

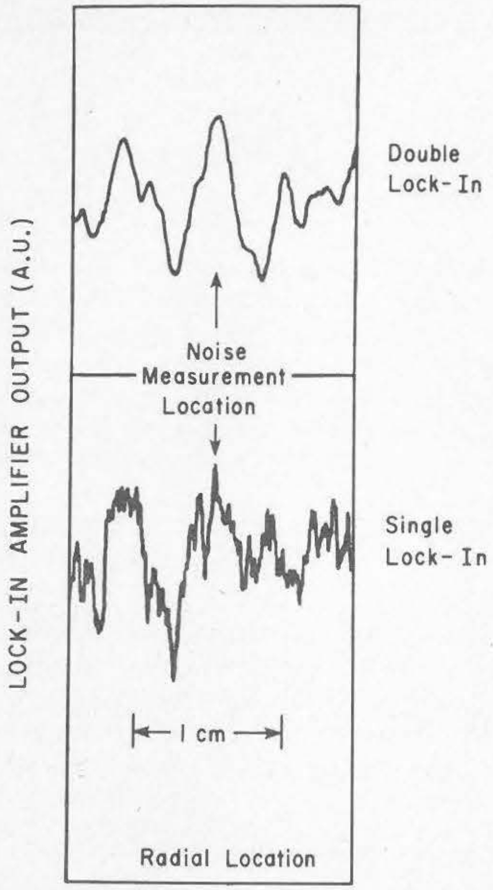

FIG. 4. Data from the demonstration experiment (reprint from Ref. 2). Interferograms of the wave in the plasma are shown for the single and double lock-in techniques, using the same final time constant. The double lock-in technique reduced noise and improved the SNR.

factor of 5.0 or more, then the second lock-in should be added, the signal modulated, and the double lock-in method employed. This, and the fact that the optimal modulation frequency is $(2\pi\tau_1)^{-1}$, are the information needed to use the double lock-in method.

II. DEMONSTRATION OF THE METHOD

To test its effectiveness, the double lock-in method was employed in an optical detection measurement. This measurement is made with a plasma diagnostic, far-infrared laser scattering, used to detect an rf wave.[2] Figure 3 shows how an antenna is driven with rf power to produce a wave. The wave Doppler shifts a far-infrared laser beam by the frequency of the driving rf oscillator. Optical mixing of the shifted beam with an unshifted local oscillator beam produces an rf signal from the detector. The frequency of the signal is the same as that of the rf power applied to the antenna, so pickup can be a problem with single lock-in detection.

By moving the laser beam slowly through the plasma, an interferogram of the wave is obtained, since the lock-in output is the signal amplitude multiplied by the sine of its phase with respect to the rf reference signal.

Figure 4 plainly illustrates that the double lock-in method can clean up a waveform, in this case the interferogram, despite rf pickup that plagues single lock-in detection. For this demonstration, $f_1 = 12.5$ MHz, $f_2 = 520$ Hz, $\tau_1 = 500$ μs, and the same final time constant $\tau_2 = 4$ s was used for both the single and double lock-ins. A measurement of noise indicated that the SNR was improved by a factor of $Q \simeq 20$ using the double lock-in method.

ACKNOWLEDGMENTS

I gratefully acknowledge helpful discussions with D. Darrow, R. Horton, D. K. Mansfield, and F. Skiff. This work was supported by U. S. Department of Energy Contract No. DE-AC02-76-CHO-3073.

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