Relationship between dust acoustic waves in two and three dimensions

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Low frequency electrostatic waves are investigated for a monolayer suspension of dust particles that are shielded by an ambient plasma of three-dimensional extension. The dispersion of the resulting dust acoustic surface waves is compared with dust acoustic waves in three dimensions and with lattice modes in two dimensions. It is found that the wave dispersion is determined by shielding of electric fields by electrons and ions on either side of the dust monolayer; this differs from previously studied cases of charged sheets in a vacuum. The phase velocity of these surface waves suggests the definition of a proper dust plasma frequency for monolayer systems. © *2006 American Institute of Physics*. [DOI: 10.1063/1.2370696]

The dust acoustic wave[1] is omnipresent in three-dimensional (3D) extended dusty plasmas and has been studied experimentally in different situations.[2–5] The special case of dust acoustic surface waves was discussed in Refs. 6 and 7, where a thin layer of dust is assumed with embedded thermal ions and electrons. This thin layer is surrounded by a plasma-free vacuum. Such a situation may be found in the dust ring of Saturn. Surface waves at the interface between an extended dusty plasma and vacuum were studied in Ref. 8.

In this Brief Communication, a different situation is studied, which is more realistic for laboratory experiments. A monolayer of identical dust particles is embedded in a 3D extended gas plasma, which provides electrostatic shielding by the ambient plasma electrons and ions. Such a situation resembles experiments on compressional waves in two-dimensional (2D) suspensions (see, for example, Refs. 9–11) with the exception that here we are not interested in the crystal structure of the suspension. Rather, we assume that the dust particles are randomly distributed in the x-y plane with an average area density of the particles $\bar{n}^{(2D)}$. In this way we ignore any tendency of the particles to arrange in a triangular lattice as can occur at low dust temperature. The corresponding volume density is $n_d=\bar{n}^{(2D)}\delta(z)$. The particles of mass m_d carry an electric charge $-Z_d e$. Dust charge fluctuations are neglected in the following.

The shielding by electrons and ions is described by a linearized Debye-Hückel model

$$n_e \approx n_{e0}\left(1+\frac{e\phi}{k_B T_e}\right), \quad n_i \approx n_{i0}\left(1-\frac{e\phi}{k_B T_i}\right), \tag{1}$$

where $n_{e0}=n_{i0}$ is the unperturbed electron and ion density far from the dust layer. Shielding by thermal ions will be assumed when we compare with Refs. 6 and 7, where similar assumptions are made. This limiting case would strictly apply when the dust layer was levitated into the quasi-neutral bulk plasma, e.g., by thermophoretic forces.

The situation for 2D suspensions in the plasma sheath, however, is different. A limiting case would be to consider a constant ion density, which may be more appropriate for describing streaming ions at high velocity. In this limit, shielding would be provided by electrons only. A more realistic approximation for shielding by nonthermal ions at Bohm velocity was discussed in Ref. 12, which resulted in an ion shielding length that is similar to the electron Debye length. Such a value for the effective ion Debye length was confirmed, for the upstream and sideways direction (with respect to the ion stream), by Lampe *et al.*[13] The downstream direction, however, is characterized by ion accumulation in the wake, as discussed earlier by different authors (see, for example, Refs. 14–18). Therefore, the assumption of isotropic shielding is not strictly valid for particles suspended in the sheath region of a discharge, where the ions are streaming with supersonic velocity. As long as these various treatments of ion shielding only lead to different values of the total Debye shielding length, their effect can be described with the present model by choosing the appropriate shielding length.

Some effects of anisotropic shielding on lattice modes, which arise from the accumulated ion charge in the wake behind the particles, were discussed in earlier papers.[16,17] Since the wake charge is only a fraction of the charge on the corresponding dust particle,[17,18] we presently neglect this additional effect for the sake of simplicity of our arguments, which primarily aim at giving a better understanding of a proper definition of the dust plasma frequency and dust acoustic velocity in two dimensions.

The steady state potential distribution $\bar{\phi}(z)$ for this charged dust plane is defined by Poisson's equation

$$\frac{\partial^2\bar{\phi}}{\partial z^2}=\frac{e}{\epsilon_0}[Z_d\bar{n}^{(2D)}\delta(z)+n_e-n_i]. \tag{2}$$

Combining Eqs. (1) and (2) results in

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$$\frac{\partial^2 \bar{\phi}}{\partial z^2} - k_D^2 \bar{\phi} = \frac{e}{\epsilon_0}[Z_d \bar{n}^{(2D)} \delta(z)], \tag{3}$$

which has the solution

$$\bar{\phi}(z) = \phi_0 \exp(-k_D|z|) \tag{4}$$

with $\phi_0 = (Z_d e n^{(2D)} \lambda_D)/(2\epsilon_0)$, the Debye wave number $k_D = [n_{i0} e^2 (T_e + T_i)/(\epsilon_0 k_B T_e T_i)]^{1/2}$, and the linearized Debye length $\lambda_D = 1/k_D$. The deviation of the electron and ion densities from their equilibrium values are

$$\delta n_e = n_e - n_{e0} = -\frac{1}{2} Z_d n^{(2D)} \frac{\lambda_D}{\lambda_{De}^2} \exp(-k_D|z|),$$
$$\delta n_i = n_i - n_{i0} = +\frac{1}{2} Z_d n^{(2D)} \frac{\lambda_D}{\lambda_{Di}^2} \exp(-k_D|z|). \tag{5}$$

Here, $\lambda_{De,Di} = [(\epsilon_0 k_B T_{e,i})/(n_{e0,i0} e^2)]^{1/2}$ are the electron and ion Debye length, respectively. Integrating the space charge density $\int_{-\infty}^{\infty} (\delta n_i - \delta n_e) e \, dz$ yields

$$[\delta n_i(0) - \delta n_e(0)] e 2\lambda_D = Z_d n^{(2D)}, \tag{6}$$

which means that a region of thickness $\Delta z = 2\lambda_D$ can be considered as an equivalent space charge layer with a homogeneous charge density $\rho = [\delta n_i(0) - \delta n_e(0)] e$ that exactly balances the dust charge in the x-y plane.

For studying wave phenomena in the x-y plane we decompose the electric potential into the equilibrium solution derived above and a wavelike perturbation $\phi(x,z,t) = \bar{\phi}(z) + \tilde{\phi}(z) \exp[i(k_x x - \omega t)]$, where we have assumed a plane wave propagating in the x direction. The particle density in the wave is $n^{(2D)}(x,t) = \bar{n}^{(2D)} + \tilde{n}^{(2D)}$. The field structure results from solving Poisson's equation in the form

$$\frac{\partial^2 \tilde{\phi}}{\partial x^2} + \frac{\partial^2 \tilde{\phi}}{\partial z^2} - k_D^2 \tilde{\phi} = \frac{Z_d e \tilde{n}^{(2D)}}{\epsilon_0} \delta(z), \tag{7}$$

which has the solution

$$\tilde{\phi}(z) = \tilde{\phi}_0 \exp\{-\sqrt{k_x^2 + k_D^2} |z|\}. \tag{8}$$

This is the potential distribution for a surface wave with a penetration depth into the ambient plasma that depends on the wave number and is shorter than the Debye length. The jump in the electric field of the wave at $z=0$ is defined by the perturbed 2D charge distribution

$$\left.\frac{\partial \tilde{\phi}}{\partial z}\right|_{z=+0} - \left.\frac{\partial \tilde{\phi}}{\partial z}\right|_{z=-0} = \frac{Z_d e \tilde{n}^{(2D)}}{\epsilon_0}. \tag{9}$$

Hence, the wave potential is related to the density fluctuation by

$$\tilde{\phi}_0 = \frac{Z_d e \tilde{n}^{(2D)}}{\epsilon_0 \sqrt{k_x^2 + k_D^2}}. \tag{10}$$

The dynamics of a dust acoustic wave enters in terms of the equation of motion for the dust velocity $\tilde{v}_d$ and the 2D equation of continuity, which read in Fourier notation

$$-i\omega \tilde{v}_d = i k_x \frac{Z_d e}{m_d} \tilde{\phi}_0,$$
$$0 = -i\omega \tilde{n}^{(2D)} + i k_x \tilde{v}_d \bar{n}^{(2D)}. \tag{11}$$

Here we have neglected effects from the dust temperature. Equations (10) and (11) yield the dispersion relation for the dust surface wave

$$\omega^2 = \frac{Z_d^2 e^2 \bar{n}^{(2D)}}{2\epsilon_0 m_d \lambda_D} \frac{k_x^2 \lambda_D^2}{\sqrt{1 + k_x^2 \lambda_D^2}}. \tag{12}$$

Note that at small k this dispersion relation is acoustic-like with $\omega \propto k$, but for large k it is dispersive. This result is identical with the dust lattice modes in the mean-field approximation for particles interacting by a Yukawa potential.[19,20] The dispersion relation for the lattice modes has previously been derived from calculations of the elastic response of particles disturbed from their equilibrium positions in the lattice.

Comparing Eq. (12) with the dust acoustic wave dispersion in three dimensions,

$$\omega^2 = (\omega_{pd}^{(3D)})^2 \frac{k_x^2 \lambda_D^2}{1 + k_x^2 \lambda_D^2}, \tag{13}$$

where $\omega_{pd}^{(3D)} = [(Z_d^2 e^2 n_d)/(\epsilon_0 m_d)]^{1/2}$ is the common dust plasma frequency at a 3D dust density n_d, we see the physical similarity of the two dispersion relations. We note that in the long-wavelength limit $(k_x^2 \lambda_D^2 \ll 1)$ the phase velocity of the dust acoustic wave reads $C_{\mathrm{DAW}}^{(3D)} = \omega_{pd}^{(3D)} \lambda_D$. In the same limit the phase velocity of the dust surface wave becomes

$$C_{\mathrm{DAW}}^{(2D)} = \omega_{pd}^{(2D)} \lambda_D, \tag{14}$$

when we introduce an effective dust plasma frequency for the 2D situation

$$\omega_{pd}^{(2D)} = \left(\frac{Z_d^2 e^2 \bar{n}^{(2D)}}{\epsilon_0 m_d (2\lambda_D)} \right)^{1/2}. \tag{15}$$

This definition is quite natural, when we remember that, in this limit, the penetration depth of the surface wave becomes the linearized Debye length. Hence, the 2D dust density defines an equivalent volume density $n_d^* = \bar{n}^{(2D)}/(2\lambda_D)$. In this sense, the compression wave of the dust particles in the x-y plane is "dressed," in the z direction, with a dynamic shielding cloud of effective thickness $2\lambda_D$. In this way, an apparent contradiction is removed, namely that a 2D particle arrangement cannot be associated with a well-defined volume density.

We compare our results to those of Yaroshenko and Verheest,[7] who treated the different case of charged dust and unequal electron and ion densities all within a vanishingly thin slab that is bounded on both sides by vacuum. They found a sound velocity

$$C^{(YV)} = \left[\frac{Z_d^2 e^2 n_d^{(2D)} k_B T_e T_i}{\epsilon_0 m_d (T_e \sigma_i + T_i \sigma_e)} \right]^{1/2}. \tag{16}$$

This expression agrees with the result in a 3D system, as can be seen by replacing the 2D densities $n_d^{(2D)}, \sigma_e, \sigma_i$ with their

3D equivalents. It is worth noting, that this expression implies a Debye length that is based on different densities $n_{e0} \neq n_{i0}$, which are needed to establish quasineutrality with the dust inside the considered infinitely thin slab. Contrariwise, our model is not based on such an assumption because the shielding occurs by the electrons and ions located outside the dust plane in a layer of equivalent thickness $2\lambda_D$.

On the other hand, we can compare with the results for 2D plasmon dispersion in the one component plasma (OCP) limit.[21] Such models were used to describe a 2D electron gas on liquid helium.[22] The OCP approximation uses a homogeneous neutralizing background of the opposite charge. The OCP model can be considered as the limiting case $\lambda_D \to \infty$. When we expand our result Eq. (12) for this limiting case, the classical result[21,22] is recovered:

$$\omega = \left(\frac{Z_d^2 e^2 n_d^{(2D)z}}{2\epsilon_0 m} k_x \right)^{1/2}. \tag{17}$$

Note that this dispersion relation is dispersive for all values of k.

This OCP limiting case is often described as a wave-number-dependent plasma frequency $\omega_p(k)$.[7,21] The difference between the two limiting cases of small and large wave numbers lies in the dependence of the penetration depth of the surface waves into the ambient medium. In our case of small wave numbers the penetration depth is limited by the Debye length whereas in the large wave number limit the penetration depth becomes proportional to the wavelength. Hence, the physical difference lies in the presence or absence of shielding. We therefore claim that the common usage to ascribe the wave-number-dependent plasma frequency to the fact that the system forms a monolayer is too simple. The deeper root of this behavior lies in the fact that the surface wave dispersion is determined by the wave fields outside the plane and their modification by plasma particles. These can be either the net charges located outside the plane as calculated in our model or the electron and ion charges confined to the dust plane in Ref. 7.

In summary, we have demonstrated that acoustic surface waves in a system with a 2D dust sheet embedded in a 3D electron-ion plasma have dispersion properties that can be reduced to the common dust acoustic wave when a proper redefinition of the dust plasma frequency is made. In the long wavelength limit the resulting dispersion is identical with dust lattice modes in the mean field approximation. The OCP limit of a wave-number-dependent plasma frequency is strictly recovered for $\lambda_D \to \infty$, i.e., when shielding becomes unimportant. The existing models for dust surface waves in thin plasma slabs had different results because of the imposed requirement that quasineutrality is established inside the plasma slab whereas the present model allows shielding by out-of-plane particles.

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[1] N. N. Rao, P. K. Shukla, and M. Y. Yu, Planet. Space Sci. **38**, 543 (1990).
[2] A. Barkan, R. L. Merlino, and N. D'Angelo, Phys. Plasmas **2**, 3563 (1995).
[3] C. Thompson, A. Barkan, N. D'Angelo, and R. L. Merlino, Phys. Plasmas **4**, 2331 (1997).
[4] V. I. Molotkov, A. P. Nefedov, V. M. Torchinskii, V. E. Fortov, and A. G. Khrapak, JETP **89**, 477 (1999).
[5] S. Ratynskaia, M. Kretschmer, S. Khrapak, R. A. Quinn, M. H. T. G. E. Morfill, A. Zobnin, A. Usachev, O. Petrov, and V. Fortov, IEEE Trans. Plasma Sci. **32**, 613 (2004).
[6] L. Stenflo, P. K. Shukla, and M. Y. Yu, Phys. Plasmas **7**, 2731 (2000).
[7] V. V. Yaroshenko and F. Verheest, Phys. Plasmas **7**, 3983 (2000).
[8] H. J. Lee, Phys. Plasmas **7**, 3818 (2000).
[9] A. Homann, A. Melzer, S. Peters, R. Madani, and A. Piel, Phys. Lett. A **242**, 173 (1998).
[10] A. Melzer, S. Nunomura, D. Samsonov, and J. Goree, Phys. Rev. E **62**, 4162 (2000).
[11] A. Piel, V. Nosenko, and J. Goree, Phys. Rev. Lett. **89**, 085004 (2002).
[12] A. Piel and A. Melzer, Plasma Phys. Controlled Fusion **44**, R1 (2002).
[13] M. Lampe, G. Joyce, G. Ganguli, and V. Gavrishchaka, Phys. Plasmas **7**, 3851 (2000).
[14] P. K. Shukla, Phys. Plasmas **1**, 1362 (1994).
[15] M. Nambu, S. V. Vladimirov, and P. K. Shukla, Phys. Lett. A **203**, 40 (1995).
[16] A. Melzer, V. Schweigert, I. Schweigert, A. Homann, S. Peters, and A. Piel, Phys. Rev. E **54**, R46 (1996).
[17] V. A. Schweigert, I. V. Schweigert, A. Melzer, A. Homann, and A. Piel, Phys. Rev. E **54**, 4155 (1996).
[18] M. Lampe, G. Joyce, and G. Ganguli, IEEE Trans. Plasma Sci. **33**, 57 (2005).
[19] D. H. E. Dubin, Phys. Plasmas **7**, 3895 (2000).
[20] G. J. Kalman, P. Hartmann, Z. Donkó, and M. Rosenberg, Phys. Rev. Lett. **92**, 065001 (2004).
[21] H. Totsuji and H. Kakeya, Phys. Rev. A **22**, 1220 (1980).
[22] P. M. Platzman, Phys. Rev. B **13**, 3197 (1976).